

MARTIN-LUTHER-UNIVERSITÄT
HALLE-WITTENBERG

Institut für Physik
Grundpraktikum



Lab course
Basic Physics

for
Polymer Materials Science

edition 2017

Preface

The lab course „Basic Physics and Physical Chemistry Lab” is part of the module „Basics of Materials and Polymer Physics”.

The subjects of the course are

- planing, performing and evaluating scientific experiments; record writing; estimation of measurement uncertainties
- working with modern measurement technique
- selected physical topics of basic physics and physical chemistry

The course consists of

- one lecture (2 h) introducing to statistics and uncertainty analysis
- one tutorial experiment (4 h, exercise in statistics and uncertainty analysis and introduction to the software Origin)
- five experiments in physics (4 h each)
- four experiments in physical chemistry (4 h each)

This booklet describes the experiments in the Physics part of the course only.

The introduction chapters are related to all general aspects of the physics lab course (safety in the lab, requirements to protocol writing and estimation of measurement uncertainties, literature and software). The main part describes shortly the physical basics of each experiment and gives detailed instructions to experimenting and evaluating the results. The questions at the end of each experiment description are mainly intended for your self check. Depending on your knowledge in Physics, you need to study the basic principles of an experiment using additional textbooks.

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Laboratory Rules

General Rules

- 1** When working in the laboratory do not endanger other persons and make sure no technical devices or experimental arrangements get damaged.
- 2** The instructions given to you by the tutor or the laboratory staff and those written in this booklet regarding the use of devices and experimental arrangements are strictly to be observed.
- 3** Please report any troubles, irregularities, damages to or malfunctions of devices as well as accidents to the tutor. You are not allowed to repair any devices by yourself!
- 4** You are to account for any damages on devices or materials caused wilfully.
- 5** Use the equipment available at your workplace only. You are not allowed to use any equipment from other workplaces.
- 6** After finishing the experiment clean up your workplace. Log off from any computer you have used.
- 7** Eating and drinking is prohibited in the laboratory rooms. The whole laboratory building is a nonsmoking area.
- 8** The use of mobile phones in the laboratory rooms is prohibited.
- 9** Laboratory courses start in time according to the timetable. If you are more than 15 minutes late the staff may enjoin you from starting an experiment.
- 10** For finishing the course successfully you need to perform all experiments. If for a serious reason (e.g. due to illness) you can not attend the laboratory course, please inform the laboratory staff and arrange an extra date to perform the missed experiment. Dates will be granted for the lecturing time of the current semester only.

Working with electrical circuits

- 11** Assemble and dismantle electrical circuits with disconnected voltage only: power supplies off, batteries not connected.
- 12** When working with measuring instruments, pay special attention to the correct polarity, to the correct measuring range and use the correct measuring inputs (danger of overloading and damaging).
- 13** You must have electrical circuits checked by the tutor before putting them into operation.
- 14** Energized systems are to be supervised permanently.
- 15** Do not touch any components carrying electric voltages. Dangerous voltages ($>42\text{V}$) are generally protected from being touched. Do not remove or short-circuit those protective equipments!
- 16** In case of an accident switch off the power immediately! (There is a yellow emergency switch in every room.) Report the accident immediately.

Working with chemicals

- 17** Work cleanly. If necessary use a funnel for transferring liquids and absorbing pads for weighting chemicals.
- 18** Any safety materials (i.g. safety goggles) given to you with the experimental accessories have to be used!
- 19** In case of accident or spilt dangerous chemicals (e.g. mercury) inform the tutor or laboratory staff immediately! Do not remove those spilt chemicals yourself!
- 20** All chemicals are stored in containers marked with a content description. Make sure you always use the correct container, especially

when pouring the chemicals back into the containers after usage.

21 After finishing the experiment, carefully rinse all used containers (except containers used for storing materials).

Working with radioactive material

22 A sealed radioactive radiation source (74 kBq Co-60) is used in the experiment O16. It is allowed to be handled by students. The radiation exposure during the experiment is 100 to 1000 times lower than during a medical X-ray examination.

23 Nevertheless avoid any needless exposure! Do not carry the radiation source in your hand if not necessary! Keep a distance of 0.5 m to the radiation source during the experiment!

24 It is prohibited to remove the radiation source from the surrounding plexiglass block.

Preventing fire

25 Place Bunsen burners or electric heaters securely so that neighbouring devices will not catch fire. Permanently supervise open fire and heaters.

26 Take care when working with flammable liquids (for example ethanol)! Keep them away from open fire!

27 In case of a fire, inform the laboratory staff immediately and take first measures to extinguish the fire.

28 You are required to know where to find the fire extinguisher, how to use it and which escape routes and exits can be used.

Procedure of a Laboratory Course

1 Preparation

The subject of your next experiment is found on the laboratory home page on the Internet or on the notice board in the corridor.

Prepare yourself at home. Study the physical basics of the experiment and prepare a protocol (see Guidelines to Protocol Writing).

2 Starting a laboratory day

Be on time. Students who are more than 15 minutes late may be excluded from performing the experiment.

You are given the experimental accessories necessary for your group on depositing a student card.

The tutor will inspect your prepared protocol and question you shortly about the physical

basics of the experiment. Students who are not prepared are not allowed to work in the laboratory at the present day.

3 Performing the Experiment

Experiments are carried out in groups of two students. Each student writes his or her own protocol.

Construct the experimental setup. **Please have electrical circuits checked by the tutor before putting them into operation.**

Perform the measurements and keep records of the results, observations and notes. Ask the tutor if you need assistance.

The tutor will check your results and authenticate it with his short signature.

4 Finishing a laboratory day

Clean up your working place. Give the accessories back and receive your students card.

The tutor must sign your records (see above).

5 Evaluation of the experiment

You will need a pocket calculator, a ruler and possibly graph paper. Graph papers (e.g. logarithmic paper) can be bought in the laboratory. You may use the computers in the lab for evaluating your data, producing graphs and printing them.

Write (or at least start writing) the evaluation of the experiment during the laboratory course. You are expected to have completed the experimental evaluation by the start of your next laboratory day.

6 Review of the record

Usually the tutor will review your record during the next laboratory day. You will be given a mark (from 1 = excellent to 5 = fail) that takes account of your preparation (your knowledge and the prepared protocol), your experimental work and your evaluation.

The mark is written into the protocol together with the full signature of the tutor.

The completely evaluated protocol must be presented not later two weeks after the date of the experiment. For every additional week your mark is downgraded by one.

7 Finishing the laboratory course

You need to completely perform 5 experiments with grade 4 or better (+ 4 in physical chemistry). The success in lab course (together with physical chemistry) is certified in the universities database.

Guidelines to Writing a Protocol

General

- Each student keeps his own record during the laboratory work. Please use a file or notebook in the size of A4.
- Use ink or ball pen for writing the record. Write immediately into the protocol, do not use extra notepad. If you've made a mistake, mark the notes or values as being wrong for a particular reason but do not erase them.
- Graphs are drawn by pencil on graph paper or printed by a computer, respectively. Label them with your name and date and include it into the protocol.

Preparation at home

- Prepare your record with the following content:
 - Date, name of the experiment and the exact task.
 - A short description of the experiment, including the formulas necessary for understanding and evaluating the task and a sketch (if applicable, i.g. an electrical circuit).
 - Prepared tables for recording the measured and (if required) the calculated values.
- This part of the record will be inspected at the beginning of your laboratory work.

Recording during the experiment

- List all devices used in the experiment.
- Keep your record clear and readable. Distinguish the different parts of the experiment clearly.
- Introduce all physical quantities with their name and symbol. In graphs and tables, write physical values with their symbol and unit of measure.
- Write all measured values (before any calculation is done) into the protocol.
- A protocol is complete, if even somebody else who did not perform the experiment can understand it and evaluate it.
- Let the tutor check and sign your record before you leave the lab.

Evaluating the results

- All calculations should be comprehensible. (It has to be clear which result was calculated from which data by which equation.)
- Graphs should be drawn clearly on graph paper using a ruler or made by computer. The axes have to be labelled with the symbol and the unit of measure.
- Estimate the experimental errors quantitatively. In some experiments, an uncertainty analysis is required.
- Write your results and the errors estimated in a whole sentence and discuss them critically. If possible, compare your results to table data.
- The protocol will be inspected on the next laboratory day. It will be certificated (grade and signature of the tutor) if the experiment was completely performed and evaluated.

Uncertainty Analysis and Statistics

Any measurement of a physical quantity is imperfect. If a quantity is measured repeatedly, the results will generally differ from each other as well as from the “true value” that is to be determined.

The objective of the uncertainty analysis is to determine the best estimation of the true value (the measurement result) and an estimation of the deviation of the result from the true value (the measurement uncertainty).

1 Definitions

Measurand:

A particular physical quantity subject to measurement; e.g. the mass m of a given

body, the voltage U of a battery at 20°C

Value (of a quantity):

Magnitude of a particular physical quantity, expressed as a number multiplied by a unit (of the quantity), e.g. mass of the body: 2.31 kg

Result of a measurement:

Value attributed to a measurand, obtained by measurement (may be calculated from several measured values). It is only an estimate of the value of the measurand (the „true value”). In general, the result should not be given without additional information about its uncertainty and the way it was obtained.

Error (of measurement):

Measurement result minus true value. The error consists of a random and a systematic part. Generally, the measurement error can not be known exactly because the true value is not known. The “error calculation” results in an estimation of the error, called uncertainty.

Random (or statistical) error:

Measurement result minus the mean that would result from an infinite number of measurements of the same measurand. The random error varies in magnitude and sign. It arises from uncontrollable variations of the experimental and environmental conditions, from physical limits of observation (e.g. noise, quantum effects) and from the limits of human senses. Random errors can be minimized by taking multiple measurements.

Systematic errors:

Mean that would result from an infinite number of measurements of the same measurand minus its true value. The systematic error is constant as long as the controllable conditions of the experiment remain constant. It arises from imperfections of the devices, calibers, measurement procedures, and from systematic changes in the experimental conditions. It may consist of a known and an unknown part. The measurement result has to be corrected by the known part of the systematic error (“corrected result”).

Uncertainty (of measurement):

Parameter u , associated with the measurement result, that characterizes the dispersion of the values that could reasonably be attributed to the measurand. The uncertainty $u(x)$ of a measurement result x may be considered as an estimation of the measurement error. The interval $x \pm u(x)$ contains with high probability the true value of the measurand.

The uncertainty is estimated on the basis

of the measured values (by statistical methods, type A) and the knowledge about the measurement system and procedure (type B).

Example

for a measured value $l = 2.041$ m:

$u(l) = 0,005$ m (**absolute uncertainty**)

$u(l)/l = 0,4$ % (**relative uncertainty**)

Complete measurement result:

Measurement result \pm uncertainty, the uncertainty given with 1 or 2 digit accuracy. Allowed notations are:

$l = 1.254$ m \pm 5 mm

$l = (1.254 \pm 0,005)$ m

$l = 1.254(5)$ m

$l = 1.254$ m and $u(l)/l = 0,4$ %

2 Determination of uncertainties

The determination of the accuracy of measurements is based on different sources of information: The statistics of measured values, manufacturer's information and certificates of measuring instruments and standards used, or simple estimate. In every case the uncertainties should be characterized by standard deviations - either calculated by statistical methods (↗type A evaluation) or estimated by other means (↗type B evaluation). To underline this fact, they are often called **standard uncertainties**.

2.1 Type A evaluation on uncertainty

A quantity x is measured n times. Because of random errors, the individual measured values x_i ($i = 1 \dots n$) will scatter around an expectation μ . In most cases the values are approximately normal distributed (see fig.1). The best approximation of μ is then the **arithmetic mean**

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i. \quad (1)$$

A measure of scattering of the values x_i is the **standard deviation** σ . The best estimation of

σ calculated on the basis of the x_i is the experimental standard deviation (sometimes denoted as sample standard deviation)

$$s = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}}. \quad (2)$$

Provided the measured values comply with a

normal distribution $\varphi(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

with the mean μ and the standard deviation σ , the probability to find a single measured value in the interval $\mu \pm \sigma$ is 68 %. For the interval $\mu \pm 2\sigma$ this probability is about 95 %.

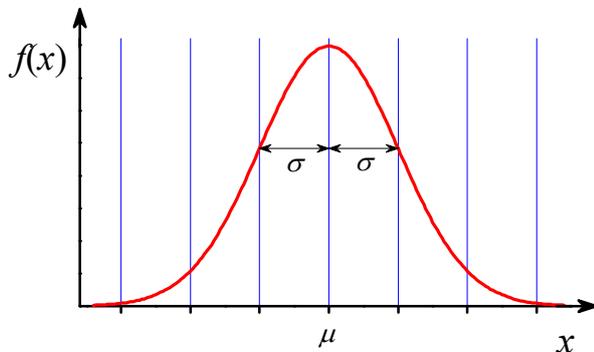


Fig. 1: Normal distribution with the mean μ and the standard deviation σ

If random error are dominating (i.e. systematic errors are negligible), the uncertainty of a single measured value is $u(x) = s$.

The mean \bar{x} of n single measurements is more accurate than one single measurement: Imagine that many series of n measurements are taken. The respective means would slightly differ from each other. They are normal distributed as well, and a measure of their scattering is the **standard deviation of means**

$$s_{\bar{x}} = \frac{s}{\sqrt{n}}. \quad (3)$$

If the result of a measurement is the mean \bar{x} of a sufficient number ($n \geq 10$) of single measured values x_i and systematic errors are negligible, the (standard) uncertainty of this

result is:

$$u(\bar{x}) = s_{\bar{x}} = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n \cdot (n-1)}}. \quad (4)$$

More examples of type A evaluation of measurement uncertainties will be given in chapter 3 (Regression Analysis).

2.2 Type B evaluation on uncertainty

If the calculation of a standard deviation is not possible (e.g. if the uncertainty is dominated by constant systematic errors or if only one measured value exists), the standard uncertainty is to be estimated on the basis of all given information.

2.2.1 Manufacturer guaranteed tolerance

The manufacturer of a measuring device usually guarantees the measurement accuracy within a certain tolerance or maximum error. Examples: 1.5 % of full scale; 0.1 % of reading + 2 digit. Sometimes the so-called accuracy class is specified which is the guaranteed accuracy in percent of full scale or of the value of the material measure.

Denoting the given tolerance of a measurand x by $t(x)$, the standard uncertainty is due to

$$u(x) = t(x)/\sqrt{3}. \quad (5)$$

Explanation:

The only known fact is that the measurement error is not larger than $t(x)$. Hence a uniform distribution with the width $2t$ can be attributed to the measurand x . The standard deviation of this distribution is $t/\sqrt{3}$.

2.2.2 Uncertainty of counter tube measurements

If a measurement consists of counting random events in a given time interval (e.g. counting the pulses of a Geiger counter for measuring radioactivity), the standard uncertainty of a result N (without considering systematic errors) is $u(N) = \sqrt{N}$.

Explanation: See experiment O16.

2.2.3 Estimation of uncertainties

When using very simple measurement devices (e.g. ruler, thermometer), there is often no information about the accuracy. In such cases the standard uncertainty (not the maximum error!) is to be estimated:

- rule of thumb for reading scales: $u(x) \approx 0.5$ division,
- Length measurement with a Vernier caliper: $u(l) = 1$ division of the vernier scale
- time measurement with a stopwatch (by hand): $\Delta T \approx 0.1$ s

2.3 Uncertainties of measurement results (uncertainty propagation)

We consider a measurement result y that is to be calculated from the measured values x_1, x_2, \dots, x_n with the respective uncertainties u_1, u_2, \dots, u_n according to $y = f(x_1, x_2, \dots, x_n)$. What is the uncertainty $u(y)$ of that measurement result?

2.3.1 The maximum uncertainty

A small change Δx_i of a value x_i would cause a change of $\Delta y_i = \frac{\partial y}{\partial x_i} \Delta x_i$ in y , where $\frac{\partial y}{\partial x_i}$ denotes the partial derivative of the function $y = f(x_1, x_2, \dots, x_n)$ with respect to x_i . If we assume the uncertainties u_i to be small compared to the values x_i , the maximum uncertainty $u(y)$ (its upper limit) is the positive sum of the influences of all u_i on y

$$u(y) = \left| \frac{\partial y}{\partial x_1} \right| u_1 + \dots + \left| \frac{\partial y}{\partial x_n} \right| u_n = \sum_{i=1}^n \left| \frac{\partial y}{\partial x_i} \right| u_i \quad (6)$$

This equation may only in simple cases (few measurands) be used for estimating the uncertainty of a measurement result.

2.3.2 GAUSS's law of propagation of uncertainty

In general the uncertainty components will not always add as in eq. (6), they may also partially compensate each other. The exact mathematical treatment of this problem by

C. F. GAUSS, assuming statistical independence of the measurands x_i , results in the „law of error propagation”

$$u_c(y) = \sqrt{\sum_{i=1}^n \left(\frac{\partial y}{\partial x_i} \right)^2 u_i^2}. \quad (7)$$

$u_c(y)$ is called **combined standard uncertainty**. Eq. (7), (9) or (11) are to be used for the evaluation of uncertainties of measurement results in the lab course in most cases.

2.3.3 Simple cases

Often the function $y = f(x_1, x_2, \dots, x_n)$ is very simple. In two special cases the calculation of the combined uncertainty according to eq. (7) can be very much simplified:

$$\text{case I:} \quad y = c_1 x_1 + c_2 x_2 \quad (8)$$

(c_1, c_2 are constants)

By inserting (8) into (7) we find:

$$u_c(y) = \sqrt{c_1^2 u_1^2 + c_2^2 u_2^2}. \quad (9)$$

$$\text{case II:} \quad y = c \cdot x_1^n \cdot x_2^m \quad (10)$$

(c real and n, m integer numbers)

Inserting (10) into (7) results in a simple equation for the relative combined uncertainty:

$$\frac{u_c(y)}{y} = \sqrt{n^2 \left(\frac{u_1}{x_1} \right)^2 + m^2 \left(\frac{u_2}{x_2} \right)^2}. \quad (11)$$

Example:

In an uniformly accelerated motion the distance d depends on time t like $d = a/2 \cdot t^2$. If d and t are measured with their corresponding uncertainties $u(d)$ and $u(t)$, and a is to be calculated, we get

$$a = 2 \cdot \frac{d}{t^2}, \quad \frac{u(a)}{a} = \sqrt{\left(\frac{u(d)}{d} \right)^2 + \left(2 \cdot \frac{u(t)}{t} \right)^2}.$$

3 Regression (fit) of a function to a series of measurements

3.1 Linear regression (linear fit)

Frequently, different measured quantities x and y are linearly related or such a relation is supposed to exist:

$$y = f(x) = a + b \cdot x. \quad (12)$$

Example:

The length of a metal rod depends on the temperature according to $l = l_0 + \alpha \cdot l_0 \cdot \Delta T$ where α is the coefficient of linear expansion, $\Delta T = T - T_0$ and l_0 is the length at $T = T_0$.

The actual task of measurement is to determine the (constant) parameters a and b . In principle, a and b can be calculated from two pairs of measured values (x, y) . In most cases, however, a whole series of measurements (n pairs of values (x_i, y_i) , $i = 1 \dots n$) is taken for verifying the linear relation. In a graphical representation the points (x_i, y_i) will scatter around a straight line, because of the unavoidable random errors. The task is now to find the straight line that „fits best” the measured points (see fig. 2).

The deviation between the measured point (x_i, y_i) and the straight line at x_i is

$$\Delta y = y_i - y(x_i) = y_i - (a + bx_i).$$

According to GAUSS's method of least squares (where it is assumed for simplification that only the values y_i are scattering), the best straight line is found by minimising the sum of squares of the Δy :

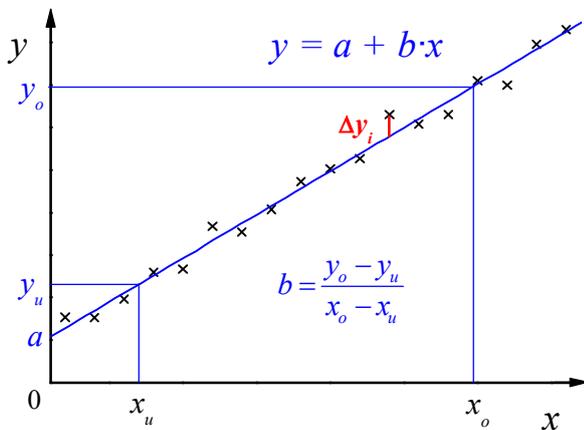


Fig. 2: Linear regression

$$F(a, b) = \sum_{i=1}^n \Delta y^2 \quad (13)$$

$$= \sum_{i=1}^n [y_i - (a + bx_i)]^2 \rightarrow \min.$$

This sum is a function of the two parameters a and b . The minimum is found by setting the partial derivatives $\partial F/\partial a$ and $\partial F/\partial b$ equal to zero. In this way we obtain

$$a = \frac{\sum x_i^2 \sum y_i - \sum x_i \sum x_i y_i}{n \sum x_i^2 - (\sum x_i)^2} \quad (14a)$$

$$= \frac{1}{n} (\sum y_i - b \sum x_i)$$

$$b = \frac{n \sum x_i y_i - \sum x_i \sum y_i}{n \sum x_i^2 - (\sum x_i)^2}, \quad (14b)$$

where all sums are taken from $i = 1$ to n .

The line defined by (12) and (14) is called the regression line.

If the statistical errors predominate the systematic errors, the uncertainties of the parameters a and b are given by their standard deviations: $u(a) = s_a$ and $u(b) = s_b$ with

$$s_a^2 = \frac{\sum \Delta y_i^2}{n-2} \frac{\sum x_i^2}{n \sum x_i^2 - (\sum x_i)^2} \quad (15a)$$

$$s_b^2 = \frac{\sum \Delta y_i^2}{n-2} \frac{n}{n \sum x_i^2 - (\sum x_i)^2}. \quad (15b)$$

3.2 Regression analysis with other functions

The method of least squares (13) is not restricted to straight lines as in eq. (12) but can be applied to all functions with any number of parameters. In general the problem cannot be solved analytically but must be solved numerically. Numerical methods for doing this „nonlinear regression analysis” are imple-

mented in many scientific software such as the programs Origin and CassyLab that are available in the Basic Laboratory. Look for the keywords *non-linear curve fit* or *free fit* in those programs.

Some nonlinear functions can be „linearized”, i.e. transformed into a linear function (for example $y = ax^b$ and $y = ae^{bx}$ by taking the logarithms). In this case, the linear fit may be performed on the transformed function. This is utilized for calculating the half value thickness of lead for gamma radiation in the lab experiment O16 (see there).

3.3 Practical hints

You don't need to keep the formulas (14) and (15) in mind, evaluations are usually done by software. You must know the basic principle of regression analysis, and the meaning of the parameters a , b , s_a and s_b .

Many scientific pocket calculators allow linear regression, check the manual of your calculator. The standard deviations s_a and s_b are usually calculated by computer software only.

In many cases (if suitable software or pocket calculator is not available or not required) it is sufficient to determine the regression

parameters a and b graphically in the following way:

Plot the measured points into a coordinate system on graph paper, draw the best fit line according to visual judgement using a transparent ruler and read the values of a and b from that line.

5 Presentation of measurement results and uncertainties

Always present the complete measurement result with absolute and relative uncertainty:

$$y \pm u(y) \quad \text{and} \quad u(y)/y. \quad (16)$$

Result and uncertainty are to be rounded correctly in the following manner:

The uncertainty $u(y)$ has to be given with an accuracy of one or two significant digits. The result has to be given with the same accuracy (the same number of decimal places) as the uncertainty.

Examples:

$$y = (431 \pm 5) \text{ mm}, \quad u(y)/y = 1.2 \%$$

$$U = (20.00 \pm 0.25) \text{ V}, \quad u(U)/U = 0.13 \%$$

$$R = 2.15 \text{ k}\Omega \pm 0.07 \text{ k}\Omega, \quad u(R)/R = 4 \%$$

English Literature on Basic Experimental Physics

Library: ULB, Zweigbibliothek Heide-Süd, Von-Danckelmann-Platz 1

General Physics

Physics for Scientists and Engineers (Physics 5e)

Paul A. Tipler, Gene P. Mosca

MODERN PHYSICS (Modern Physics 4e)

Paul A. Tipler and Ralph A. Llewellyn

Physics for scientists and engineers

Paul M. Fishbane. - 2. ed., extended. - Upper Saddle River, NJ : Prentice Hall, c 1996

Physics for scientists and engineers

Douglas C. Giancoli. - 2. ed.. - Englewood Cliffs, N. J. : Prentice Hall, 1988

Thermodynamics for engineers

Kau-Fui Vincent Wong. - Boca Raton, Fla [u.a.] : CRC Press, c200

Basic optics for electrical engineers

Clint D. Harper. - Bellingham, Wash. : SPIE, 1997

Laboratory Work and Error Analysis

Practical Physics

G. L. Squires. - Cambridge University Press, 2001

The art of experimental physics

Daryl W. Preston. - New York [u.a.] : Wiley, 1991

Experimentation and uncertainty analysis for engineers

Hugh W. Coleman. - New York [u.a.] : Wiley, 1989

GUM 2008: Guide to the Expression of Uncertainty in Measurement

<http://www.bipm.org/en/publications/guides/gum.html>

Other Literature

Math refresher for scientists and engineers

John R. Fanchi. - 2. ed. - New York, NY [u.a.] : J. Wiley, 2000

Physical properties of materials for engineers

Daniel D. Pollock. - 2. ed. - Boca Raton, Fla. [u.a.] : CRC Press, 1993

Software in the Basic Laboratory

All software used in the basic lab may be freely used (with some limitations) on private computers.

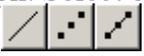
Programs made by the educational systems manufacturer LD Didactic GmbH can be downloaded from their website <http://www.ld-didactic.com>. These are:

- CASSY Lab (used in W12 - humidity)
- X-Ray Apparatus (used in O22 - x-ray methods)
- Digital Counter (used in O16 - radioactivity)

For evaluating and plotting experimental results, the professional data visualisation and analysis software Origin 2016-2017 is available on all computers in the lab and in the students computer pools. The university owns a campus licence that allows the use even on private computers, provided there is a VPN connection to the university network (ask the staff in the lab for technical details). Alternatively, there is the free Origin clone SciDAVis (<http://scidavis.sourceforge.net/>). This program runs on Windows, Linux and Mac OS and can read and write Origin files up to version 7.5 (unfortunately not compatible with the workbooks introduced in Origin 8).

Short introduction to ORIGIN

1. General aspects

- All data, calculations and graphs are saved together in a project file. An empty project (at program start) contains only the workbook **Book1** with one x and one y column for data input. More columns can be added with **Add New Column** or , more Workbooks with **File - New** or .
- The fastest way to get a graph: Select one or more y columns and choose **Plot** from the menu or click one of the buttons .
- All objects (e.g. column names and labels, axis labels, curve styles, legend) may be edited by double-clicking them.
- If the program starts in German language, switch to English via **Hilfe - Sprache ändern...** and restart the program.

2. Workbooks

- The input of data works like in MS Excel. Get more columns with .
- Give meaningful names to columns: Double-click the column header and enter **Long Name** and **Units**. These are automatically used in graphs and legends.
- Denominating a column as x or y: Mark the column and click  or right-click it and choose **Set As**.

- Calculations with columns: Mark the column, right-click it and choose **Set Column Values...**

Syntax:	column A - column B	write as	col(A) - col(B)
	$a b / (c + d)$		$a * b / (c + d)$
	x^2		x^2
	\sqrt{x}		sqrt(x)
	e^x		exp(x)
	π		pi

3. Graphs

- In Origin a coordinate system is called a layer. One graph may contain one or more layers.
- Refurbishing a graph: Double-click all things you want to change.
- Adding a curve to an existing graph:
Way 1: Select the columns to plot in the worksheet, grab it on the right edge with the mouse and drag it into the graph.
Way 2: Double-click the layer icon  in the upper left corner of the graph and follow the dialogue.
- Adding a coordinate system or an axis to an existing graph: Choose **Edit - New Layer(Axes)** or click one of the buttons .
- Add a legend or refresh an existing legend: Choose **Graph - New Legend** or press .
- Write text to your graph with the  tool. Use the format toolbar for Greek and indices.
- Read values from a graph with the Screen Reader or Data Reader  .
- Drawing smooth curves through measurement points: Double-click the curve, choose the plot type **Line+Symbol** and select **Line - Connect - Spline** or **B-Spline**.
- Linear regression: Choose **Analysis - Fit Linear**. If there are more than one curve in the graph, select the right one in the **Data** menu before. If only a part of the curve is to be fitted, define the range before with the Data Selector  and the Mask Tool .

4. Printing graphs and worksheets

- Check your graphs before printing (or have it checked by the tutor). Print only once for each student. Wasting paper costs money and pollutes the environment. Do not print very large worksheets (many pages).
- Combine several graphs and worksheets on one layout page: Choose **File - New... Layout** or click . Right-click the layout to add graphs and worksheets.
-  prints a graph or worksheet immediately on an A4 sheet.

RLC oscillator

E 40

1 Task

1.1 Learn how an oscilloscope is operated for measuring voltage, time and frequency.

1.2 Measure the natural frequency f_0 and the attenuation α of a *RLC*-oscillator and record the resonance curve.

2 Physical Basics

2.1 The oscilloscope (or scope) is a very versatile instrument with many applications, which allows the visualisation of rapidly changing electrical signals on a screen (see fig.3 next page). In most applications, the vertical (Y) axis of the screen represents a voltage and the horizontal (X) axis the time. X may also represent another voltage. The intensity or brightness of the display is sometimes called the Z axis.

Usually, an oscilloscope is capable of showing (at least) two signals at the same time, which is well-suited for testing electric circuits by comparing input and output signals.

A classical (analogue) oscilloscope basically consists of a cathode ray tube (CRT, sometimes also called BROWN's tube). In the CRT, an electron beam is formed by an electrode system. The electron beam is passing through

a deflection unit, consisting of two pairs of metal plates arranged in horizontal and vertical direction. When a voltage is applied to a plate pair, the electron beam is deflected by the electric field. The beam then hits the screen of the CRT, where it causes the luminescent coating to glow. Since the deflection angle is proportional to the applied voltage, the magnitude of that voltage can be measured on the screen. In modern digital oscilloscopes the voltages are measured by very fast A/D converters and the results are shown on a computer screen. Operation, look and feel is very similar to classical devices.

For measuring voltages over a wide range (from mV to V), the oscilloscope is equipped with adjustable amplifiers. The amplification is selected with the knob **VOLTS/DIV** (see fig.3) that controls the Y scaling factor in Volts per grid unit (1 cm).

For drawing an $U(t)$ graph (voltage vs. time), a so-called sweep voltage is applied to the X-plate pair. During a certain time period (the rise time or sweep time), this voltage constantly increases and hence guides the electron beam in the x-direction over the screen with a constant rate. Subsequently, the voltage drops to zero, and the beam therefore returns to its starting position. The voltage to be measured is applied to the Y-plate pair.

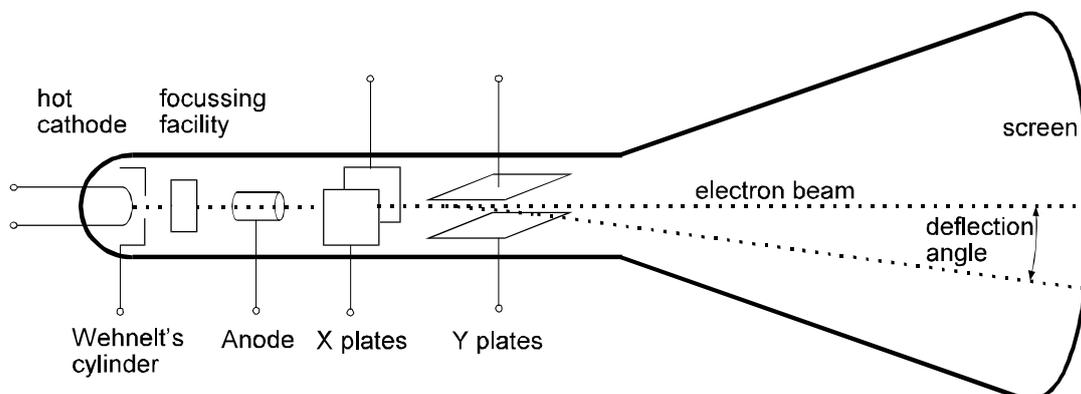


Fig.1: Cathode ray tube (CRT)

During the rise time, the electron beam writes the graph of the function $U(t)$ on the screen. This graph is refreshed every new period of the sweep voltage.

The time base knob (**TIME/DIV**) allows you to change the sweep time over a wide range ($2\text{s} \dots 0.1\mu\text{s}$). With this knob you select the X scaling factor which is the time for a horizontal deflection by one grid unit (1 cm).

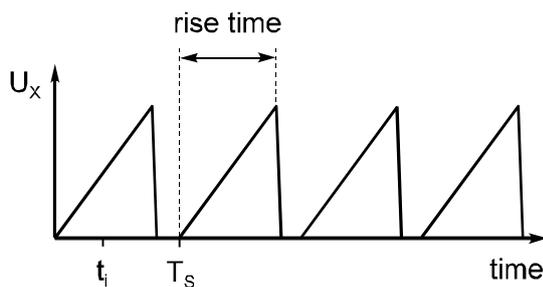


Fig.2: Sweep voltage for X deflection with the period T_s

For obtaining a stagnant pattern from periodically changing signals, one period of the sweep voltage must be an integer multiple of one period of the measured signal. This synchronisation is performed by a component called the trigger. The sweep pulse is triggered when the signal voltage reaches a certain level (which can be controlled by the **LEVEL** knob).

Like most oscilloscopes the HM303-6 is equipped with two identical input channels. Additional controls are provided for switching between one Y-t graph (**CH I/II**), two Y-t graphs (**DUAL**) or **X-Y** graph. In X-Y mode the time deflection is disabled, the input CH I is applied to the X-plates and the input CH II to the Y-plates.

The front panel of the oscilloscope is clearly organised: there are groups of knobs and buttons responsible for channel I (Y1 or X) and channel II (Y2) input, for the time deflection, the trigger control, and for the operation mode. The Y (volts) and X (time) deflection are adjusted by a rotary switch and a continuous rotary knob. The labels on the rotary switch are only valid when the rotary knob is in its rightmost position (position **CAL**). The switch **AC-GND-DC** at the signal inputs selects the coupling of the measured signal to the pre-amplifier: for direct coupling (DC), the entire signal is measured, for capacitive coupling (AC), only the alternating voltage part is measured, and for position GND, the input is grounded and separated from the signal.

Coaxial cables and BNC plugs/sockets are used for connecting a signal to the oscilloscope (this is important when high frequent signals are investigated). The core lead

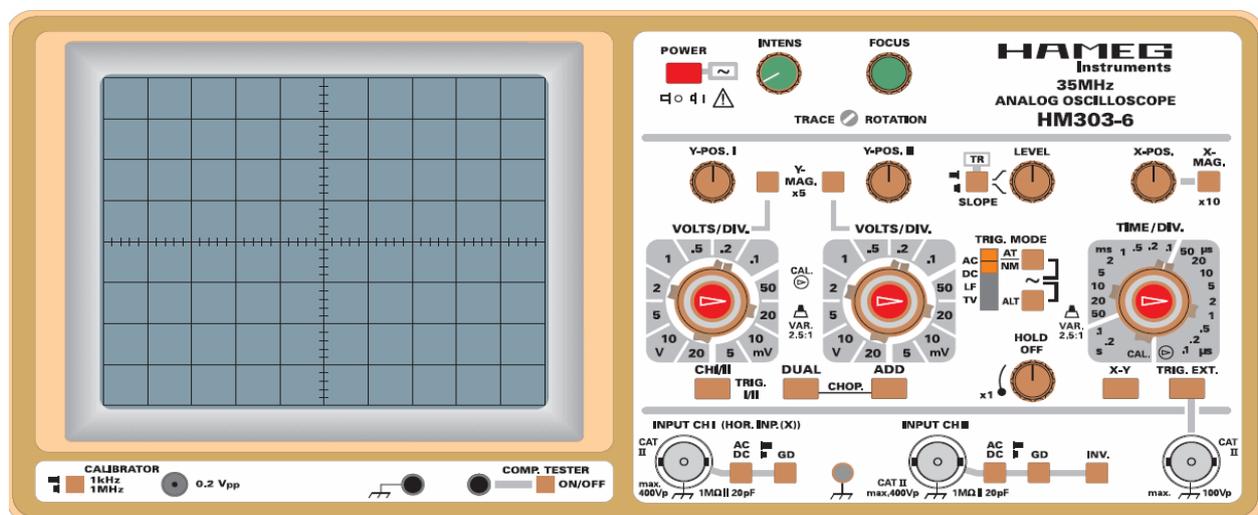


Fig.3: Front panel of the HM303-6 oscilloscope. The square divisions on the screen are 1 cm \times 1 cm

carries the signal and the metal sheath (the shield) is usually connected to ground. When a coaxial cable is connected to a normal (bifilar) cable, the core is connected to the red lead and the shield to the black lead.

Pay attention that the shield of all BNC sockets at the oscilloscope is internally connected to the protective earth conductor!

2.2 A capacitance C and an inductance L in a loop form an ideal LC circuit or LC resonator, see fig.4. If energy is brought into the circuit (e.g. the capacitor is charged with the charge q), electric oscillations occur. The electric energy flows back and forth between the inductance and the capacitance.

According to the Kirchhoff's loop rule, the voltage drop U at the inductance and at the capacitance is equal at any time. Using $C=q/U$, the discharge current of the capacitance is

$$I = -\frac{dq}{dt} = -C \cdot \frac{dU}{dt}. \quad (1)$$

The voltage drop at the capacitance is

$$U = L \cdot \frac{dI}{dt}. \quad (2)$$

By differentiating (1) with respect to t , replacing dI/dt in (2) and rearranging we get

$$\frac{d^2U}{dt^2} + \frac{1}{LC}U = 0. \quad (3)$$

This is the differential equation of an undamped harmonic oscillator being solved by

$$U = U_m \cdot \cos(\omega_0 t + \varphi) \quad (4)$$

with the angular frequency

$$\omega_0 = \frac{1}{\sqrt{L \cdot C}}. \quad (5)$$

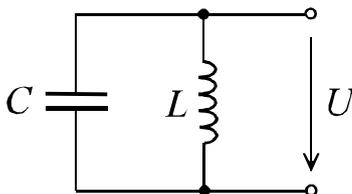


Fig.4: Ideal LC resonator

The voltage oscillates with the natural frequency ω_0 defined by the quantities L and C . The peak voltage U_m depends on the starting conditions.

In a real circuit the resistance of the wires and the coil is not zero. A real electric oscillator is a RLC resonator. At the resistance R electric energy is dissipated into heat. Therefore the peak voltage decreases, the oscillations are damped.

Figure 5 shows a RLC resonator being sinusoidally driven by an AC generator. In this circuit, two resistances have to be considered separately: The serial resistance R_s (the sum of the inner resistance of the coil and the resistance of the wires), and the parallel resistance R_p (the sum of the inner resistance of the generator and the isolation resistance of the capacitor). The higher R_s and the lower R_p , the stronger the damping of the oscillator. With the directions of the three currents and the voltage as shown in fig.5 we can write:

$$I_1 + I_2 + I_3 = 0 \quad \text{with} \quad (6)$$

$$I_1 = \frac{U - U_G}{R_p}$$

$$I_2 = C \cdot \frac{dU}{dt}$$

and

$$U = I_3 R_s + L \cdot \frac{dI_3}{dt}. \quad (7)$$

For simplification, we only consider the case

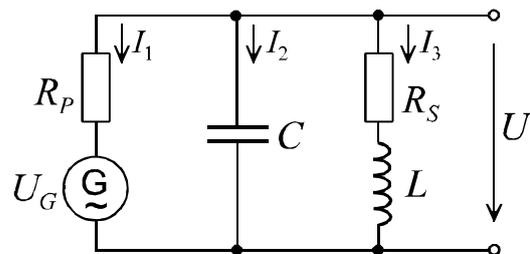


Fig.5: Real RLC oscillator with AC generator G for driving the circuit to forced oscillations

$R_S = 0$ in the following. This is approximately true in the experiment. The circuit is then called a parallel RLC circuit.

Let the generator voltage be

$$U_G = U_0 \cdot \cos \omega t . \quad (8)$$

Solving equation (6) for I_3 , differentiating it with respect to t and replacing dI_3/dt in (7) with the result yields after rearrangement

$$\frac{d^2U}{dt^2} + \frac{1}{R_p C} \frac{dU}{dt} + \frac{1}{LC} U = \frac{\omega}{R_p C} U_0 \cos \omega t . \quad (9)$$

This is the well-known differential equation for forced damped oscillations

$$\frac{d^2U}{dt^2} + 2\alpha \frac{dU}{dt} + \omega_0^2 U = K \cdot e^{i\omega t} \quad (10)$$

with the natural frequency ω_0 according to eq. (5), the attenuation

$$\alpha = \frac{1}{2R_p C} \quad (11)$$

and the factor

$$K = \frac{\omega}{R_p C} U_0 . \quad (12)$$

The complex term $e^{i\omega t}$ in (10) instead of $\cos \omega t$ just simplifies the solution of eq. (9); only the real part has a physical meaning.

The solution of the inhomogeneous linear differential equation (9) is the sum of the general solution of the related homogeneous

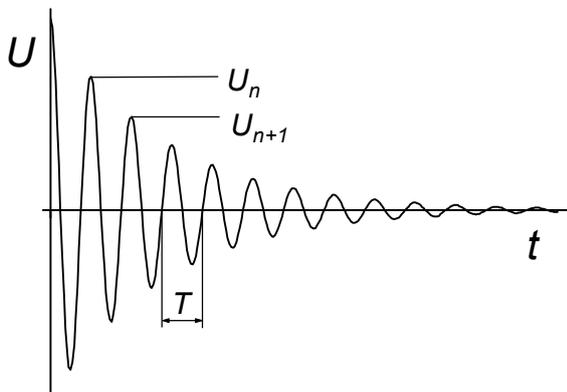


Fig. 6: Damped oscillations according to (13)

equation (natural oscillation; $K = 0$) and the particular solution of the inhomogeneous equation (forced oscillation).

For our experiment, we only consider the case $\omega_0 > \alpha$ (underdamped RLC circuit). In that case, the solution of the homogeneous eq. is

$$U = U_m \cdot e^{-\alpha t} \cdot \cos \omega' t , \quad (13)$$

$$\omega' = \sqrt{\omega_0^2 - \alpha^2} .$$

This is the damped natural oscillation that decays exponentially with the time constant $1/\alpha$ (see fig.6).

You can easily determine the attenuation α in the experiment via the logarithmic decrement

$$\delta = \ln \frac{U_n}{U_{n+1}} = \frac{1}{n} \ln \frac{U_1}{U_n} = \alpha \cdot T . \quad (14)$$

The particular solution of the inhomogeneous eq.(10) we are looking for is the steady-state solution for $\alpha t \gg 1$. We find it with the approach

$$U(t) = A \cdot e^{i(\omega t + \varphi)} . \quad (15)$$

By inserting (15) into (10) and taking into account that A and K are real, we get

$$A = \frac{K}{\sqrt{(\omega^2 - \omega_0^2)^2 + (2\alpha\omega)^2}} , \quad (16)$$

$$\varphi = \arctan \frac{2\alpha\omega}{\omega^2 - \omega_0^2} .$$

We finally replace the angular frequencies ω by the frequencies f that are more common in everyday life and K by (12):

$$A = \frac{U_0 \cdot f / (R_p C)}{\sqrt{4\pi^2 (f^2 - f_0^2)^2 + 4\alpha^2 f^2}} . \quad (17)$$

This is the amplitude (peak value) of the voltage measured at the RLC circuit when it is driven by U_G . If the driving frequency f is equal to the natural frequency f_0 , the ampli-

tude is maximum, i.e. resonance occurs.

The Q factor (quality factor) of a resonator is

$$Q = \frac{f_0}{\Delta f}, \quad \Delta f = f_u - f_l \quad (18)$$

with the upper and lower cut-off frequencies f_u and f_l . These are the frequencies where

$A = A_{\max}/\sqrt{2}$. For a parallel RLC circuit, the quality factor is

$$Q = R_P \sqrt{\frac{C}{L}}. \quad (19)$$

3 Experimental setup

3.0 Devices

- plug-in board 20×30 cm
- plug-in elements: LC resonator circuit, 2 resistors, bridging plug
- waveform generator HM8130
- oscilloscope HM303-6 with manual
- 3 BNC cables, BNC tee connector

3.1 The circuit is assembled with the given components on the plug-in board.

The data of the components are:

$L = 10$ mH, $C = 100$ nF, $R_1 = 1.0$ k Ω , $R_2 = 5.6$ k Ω , uncertainty of all values: ± 5 %.

The voltage amplitudes displayed at the waveform generator and measured with the oscilloscope are peak-peak values $U_{pp} = 2U_0$, if U_0 denominates the maximum or peak value according to equation (8).

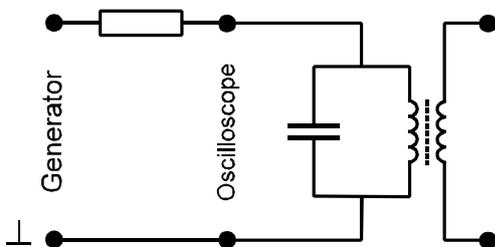


Fig. 7: Measurement circuit. The two connectors on the right are not in use.

4 Experimental procedure

4.1 Examine the oscilloscope and discuss it with the tutor.

4.2 Assemble the circuit according to fig.7, with the resistor R_1 (1 k Ω), using the input **CH I** at the oscilloscope. Additionally, connect the generator output to the input **CH II** with the help of the tee connector.

For observing the damped oscillations as shown in fig.6, apply a square wave voltage to the resonator circuit, with a large period compared to the decay time of the resonator. About 100 Hz and 3 V (peak-peak) are favourable values. Observe both the oscillator voltage and the generator voltage, using the last one as trigger source. Exchange R_1 with R_2 and see the differences. If you have a camera or mobile phone, you may take pictures.

Measure the oscillation period and the height of up to five consecutive maxima for both resistors.

For recording the resonance curves, switch the waveform of the generator to sine; the amplitude remains at 3 V. Measure the voltage at the resonator in the frequency range from 1 kHz to 15 kHz for both resistors.

Hints: At first, search for the resonance frequency f_0 . Take about 20 measurements for each curve. Choose smaller steps between the points in the vicinity of f_0 and larger steps at the ends of the frequency range.

Estimate the phase shift φ between the oscillator voltage and the driving voltage at 1 kHz, at resonance and at 15 kHz.

5 Evaluation

5.2 Calculate the natural frequency f_0 and the two attenuations α from the given data of the components.

Calculate the natural frequency, the logarithmic decrements and the attenuations from the

measured oscillation periods and the heights of the oscillation maxima with the help of eq.(14).

Plot the two resonance curves (U versus f) in one diagram. Compare the resonance frequencies obtained from the curves with the natural frequencies obtained from the damped oscillations and from the component data.

Determine the upper and the lower cut-off frequency f_u and f_l and the bandwidth Δf for each resonance curve. Calculate the Q factors of the two RLC circuits according to (18) and (19) and compare the results.

Facultative task for students with high skills

in computing (Origin): Determine α and f_0 by non-linear least squares fitting of eq.(17) to the measured data. Use $U_0/(R_p C)$ as a third free parameter in your fitting model.

6 Questions

6.1 What is an oscilloscope used for? Which quantities can be measured with it?

6.2 Explain how R , L and C behave in an AC circuit.

6.3 What is and which properties have parallel and serial RLC resonators?

Viscosity (falling ball viscometer)

M 14

1 Task

Determine the viscosity of ricinus oil as a function of temperature using a HÖPPLER viscometer (falling ball method).

2 Physical Basis

In real liquids and gases there are interactional forces between the molecules of one substance called “cohesion”, and between the molecules of different substances at an interface (i.e. a liquid and the wall of the container) called “adhesion”. When considering ideal liquids or gases, these forces are neglected.

If a real liquid flows through an inelastic tube with a circular cross-section, in the case of laminar flow a parabolic flow profile (that is flow velocity versus diameter) appears as shown in fig.1. Caused by the forces of adhesion, the liquid adheres at the wall while in the centre the velocity is maximum. For mathematical modelling, the flow is considered as concentric cylindrical layers moving with small velocity differences against each other. In between the layers friction occurs caused by cohesion. The viscosity η is a measure for this so-called inner friction.

For a long thin tube of length l and radius r , the volume flow (volumetric current) at given pressure difference Δp between the ends of the tube is described by the Hagen-Poiseuille law:

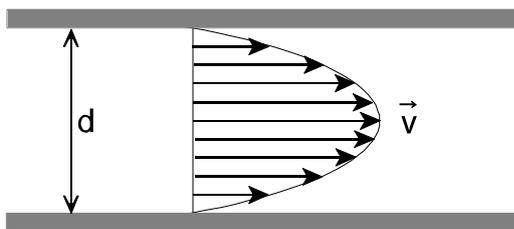


Fig.1: Laminar flow in a tube

$$\frac{V}{t} = \frac{\pi r^4 \Delta p}{8 \eta l}. \quad (1)$$

The unusual strong dependency on the radius is caused by the inhomogeneous velocity distribution (fig.1).

A liquid, the viscosity of which does not depend on the flow velocity, is called a NEWTONIAN liquid (also NEWTONIAN fluid or ideal viscous liquid). Most of the homogeneous liquids (i.e. water, oil) behave like this, while fluids consisting of different phases (i.e. ketchup, printing ink, blood) are non-NEWTONIAN fluids.

On a spherically shaped body (radius r , density ρ_1) sinking within an ideal viscous liquid (density ρ_2), the weight force F_w , the buoyancy force F_b and the friction force F_f are acting:

$$F_w = F_b + F_f. \quad (2)$$

According to the principle of ARCHIMEDES, the buoyancy force is equal to the weight of the liquid that is displaced by the body:

$$F_b = \frac{4}{3} \pi r^3 \rho_2 g, \quad (3)$$

and the weight F_w is

$$F_w = \frac{4}{3} \pi r^3 \rho_1 g, \quad (4)$$

where g is the acceleration of fall.

Because the friction force depends on the velocity according to STOKES' law

$$F_f = 6 \pi \eta r v, \quad (5)$$

after a short time of accelerated movement a steady state is reached (if $F_f = F_w - F_b$) with a constant falling speed. From eq. (1) follows:

$$\frac{4}{3} \pi r^3 \rho_1 g = 6 \pi \eta r v + \frac{4}{3} \pi r^3 \rho_2 g \quad (6)$$

and

$$6\pi\eta r v = \frac{4}{3}\pi r^3 g (\rho_1 - \rho_2). \quad (7)$$

With eq. (6), the viscosity of a Newtonian liquid can be determined from the equilibrium velocity of a sphere falling within the infinite liquid. Replacing the velocity by the elapsed time t for moving a given distance s ($v = s/t$), we obtain

$$\eta = \frac{2}{9} \frac{r^2}{s} (\rho_1 - \rho_2) g t. \quad (8)$$

All invariant quantities are now combined to the so-called geometry factor K :

$$\eta = K \cdot (\rho_1 - \rho_2) \cdot t. \quad (9)$$

In a HÖPPLER viscometer the ball does not fall within an infinite liquid but in a tube with a diameter only slightly larger than that of the ball and tilted 10° against vertical. In this case the geometry factor is not calculated but determined experimentally.

The viscous behaviour of a liquid (and some other properties too) can be understood with the help of the interchange model. The particles (atoms or molecules) are held in their places by bonding forces. They perform thermal oscillations around their places with constantly changing kinetic energy (by interaction with their neighbours). For moving to a nearby place, they have to overcome a potential barrier. That means, their kinetic energy must be higher than a certain excitation energy E_E . The velocity of thermally oscillating particles is MAXWELL-distributed. Therefore the number of place interchanges j must be

$$j \sim e^{-\frac{E_E}{kT}} \quad (10)$$

(\sim means proportional, k is BOLTZMANN's constant, T the temperature).

A force applied to the liquid from outside causes a potential gradient. Place interchanges in the direction of that potential gradient are favoured - layers of the liquid are

displaced against each other. The higher j is, the faster the displacement is. Therefore the viscosity behaves approximately like

$$\eta = \eta_0 \cdot e^{\frac{E_E}{kT}}. \quad (11)$$

In contrast to liquids, in gases the viscosity increases with rising temperature according to $\eta \sim \sqrt{T}$.

3 Experimental setup

3.0 Equipment:

- HÖPPLER viscometer
- circulator thermostat
- 2 stopwatches

3.1 The falling ball viscometer is a precision instrument. It consists of revolvable falling tube filled with the liquid to be investigated. On the tube are three cylindric measuring marks. The distance between the upper and the lower mark is 100 mm and between upper and middle mark 50 mm. The falling tube is surrounded by a water-bath, its temperature is controlled by the circulator. The instrument can be rotated by 180° into the measuring position (thermometer upright) and the roll-back position, respectively, where it locks. In your lab work you can measure with sufficient accuracy in both directions.

At the upper end of the falling tube is a hollow stopper that contains some air. This is needed for preventing too high pressures when the liquid expands at elevated temperatures. The stopper should be always above (i.e. device in measuring position) when the temperature is changed and when the experiment is finished.

The viscometer is delivered together with a set of 6 balls for different viscosity ranges. In this experiment, ball No. 4 is used. The exact value of geometry factor (the ball constant) K is given in a test certificate provided by the manufacturer for each device separately.

4 Experimental procedure

Study the manuals of the viscometer and the circulator. Do not power the heater of the circulator before setting the working temperature to a low value (cooling the bath circulator down again requires much time).

The viscosity is to be measured at five different temperatures between room temperature and 50°C. At first, align the viscometer exactly horizontal with the help of the water level on the base. Before the first measurement, let the ball fall through the tube once to ensure that the liquid is mixed well.

For determining the viscosity, you have to measure the time it takes for the ball to cover the distance between the upper and the lower measuring mark. Both students shall measure this time independently: The first student starts and stops his watch when the ball touches the measuring marks, and the second student starts and stops his watch when the ball just leaves the marks. The values are to be taken four times at each temperature (2 students get independent measurements).

If a big air bubble obstructs the falling of the ball please ask your supervisor for help. You are not allowed to open the viscometer by yourself.

The experiment is started at room temperature. Power the circulator (after a while the display should indicate **OFF**) and set the working temperature **T1** to 20°C or any temperature below room temperature. Then activate the pump by pressing the start/stop key. Observe the thermometer in the viscometer (not on the circulator!). If the temperature remains constant, wait about five more minutes for the temperature of the ricinus oil to take the same value. During the waiting time, let the ball run through the tube once. Then measure the fall time four times.

Increase the temperature step by step (four steps of 6...8 K) until 50°C is reached. After each step wait for equilibration of temperature as described above and measure the fall time.

5 Evaluation

Calculate the viscosity from the average of the measured fall times according to eq. (8) and plot it graphically as a function of temperature.

The density of ricinus oil is $\rho_2 = 0.96 \text{ g/cm}^3$.

The density ρ_1 of the ball and the geometry factor K are to be taken from the test certificate that is found at the lab station.

Discuss the experimental errors quantitatively.

Plot $\ln(\eta)$ versus $1000/T$ (this is a very common plot type for thermal excited physical and chemical processes). According to eq. (11) this should result in a straight line. Calculate the excitation energy E_E from the slope of that line. Express E_E in eV (electron volt) and in kJ/mol and compare it with typical ionisation energies and reaction enthalpies.

6 Questions

6.1 How do real liquids differ from ideal liquids?

6.2 What is inner friction? How can it be measured?

6.3 How does inner friction influence the flow of a liquid through a tube?

Diffraction Spectrometer

O 6

1 Tasks

- 1.1 Adjust a diffraction spectrometer.
- 1.2 Determine the wavelengths of the Helium spectral lines.

2 Physical Basis

Diffraction means the deviation from the way in which light propagates according to the laws of geometrical optics. It can be understood only if light is considered a wave. Diffraction always appears when the free propagation of a wave is obstructed, as for example by an edge, by a single slit or by many slits (grating).

Diffraction is usually explained by means of the Huygens-Fresnel principle. According to this, each point of a wave front is the origin of a new elementary (spherical) wavelet. The sum of these elementary wavelets forms the new wave front. If a plane light wave hits an obstacle, the wave front behind cannot be formed completely because the elementary waves from the opaque regions of the obstacle are missing. At an edge, the elementary waves also propagate as spherical wavelets into the geometrical shadow space (see fig.1). Fig.2 shows an optical diffraction grating

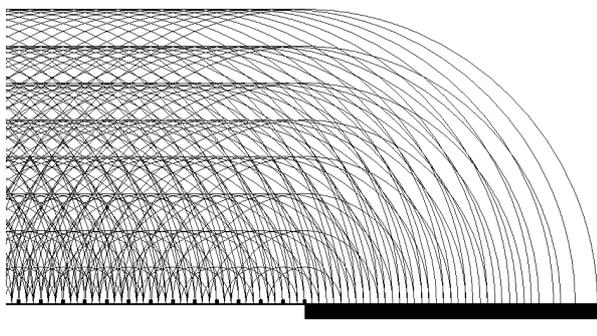


Fig.1: Diffraction of a plane wave on a edge. Construction after Huygens-Fresnel.

which is a plane two-dimensional periodical arrangement of transparent (permeable to light) and opaque zones. The distance between these zones (the grating constant b) is of the order of magnitude of the light wavelength. If a plane wave reaches the grating, circular wavelets will appear behind each slit. While propagating, they will meet wavelets from the neighbouring slits. The superposition of waves (i.e. the summation of their amplitudes) is called interference. On observation from a far distance, maxima and minima of light intensity occur by destructive and constructive interference, respectively. For simplification, each slit in fig.2 is considered to be the origin of only one elementary wavelet. The path difference δ between the wavelets coming from neighbouring slits is

$$\delta = b \cdot \sin \varphi. \quad (1)$$

Constructive interference occurs for a path difference

$$\delta = k \cdot \lambda \quad (2)$$

and destructive interference for a path difference of

$$\delta = (2k + 1) \cdot \frac{\lambda}{2} \quad (3)$$

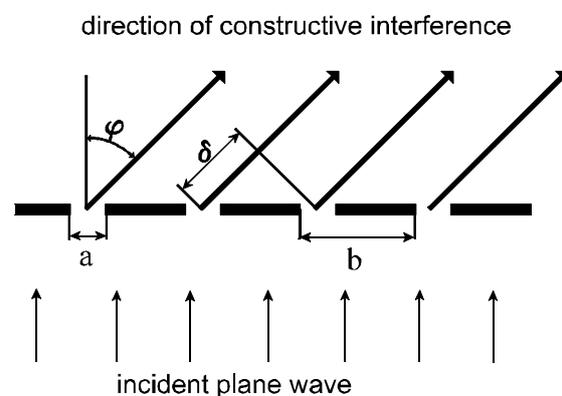


Fig.2: Calculation of the path difference δ of diffracted light on a grating.

a : width of a slit, b : grating constant, φ : angle of diffraction

where $k = 0, 1, 2, 3, \dots$ is called the diffraction order. The undiffracted (straight on going) light is referred to as zeroth diffraction order ($k=0$).

From the equations (1) and (2) the angle φ_k of the diffraction maxima follows:

$$\sin \varphi_k = \frac{k \cdot \lambda}{b} \quad (4)$$

The more wavelets constructively interfere in this direction, the more intensive and sharp these maxima are. This implies that the number of slits involved should be large.

Equation (4) shows that the diffraction angle depends on the wavelength. Thus, white light can be decomposed into its spectral colours with a grating. By measuring the diffraction angle the wavelength of light can be determined:

$$\lambda = \frac{b \cdot \sin \varphi_k}{k}. \quad (5)$$

The capability of a spectrometer is characterised by its resolution

$$R = \frac{\lambda}{\Delta \lambda} \quad (6)$$

where $\Delta \lambda$ is the smallest resolvable wavelength difference. The theoretical resolution of a diffraction grating is

$$R = N \cdot k \quad (7)$$

where N is the total number of slits illuminated and contributing to the interference pattern, and k is the diffraction order.

The principle of a diffraction spectrometer is shown in Fig.3. The slit is located in the focal plane of the collimator lens, so that the grating is illuminated by parallel light. The

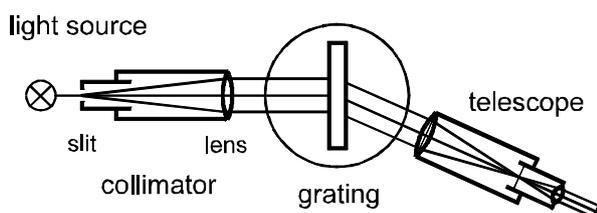


Fig.3: sketch of a diffraction spectrometer

parallel diffracted light (that apparently comes from infinity) is either observed by a telescope or focussed by a lens to a photographic plate or a CCD sensor.

3 Experimental setup

3.0 Devices:

- Goniometer with collimator and telescope
- diffraction grating in a slide, 140 lines/mm (exact grating constant on the slide)
- Helium-lamp with power supply
- Hand lamp with transformer
- Auxiliary mirror.

3.1 The experimental arrangement is according to fig.3. The goniometer ERG3 is used for measuring the diffraction angle φ . It consists of the collimator with slit and lens, a rotatable table with the grating, a moveable telescope and an equipment for measuring angles with an accuracy of 1' (angular minute). The He-lamp is placed in front of the slit. The slit width, the collimator (distance between slit and lens) and the telescope can be adjusted.

4 Experimental procedure

At first, learn how the goniometer is operated. Note the grating constant to your protocol that is written on the grating.

4.1 Adjustment of the spectrometer:

The goal is to lighten the grating with an exactly perpendicular incident beam of parallel light and to see a sharp image of the slit and of the hair cross (reticle) in the telescope.

Telescope: Make sure the position of the filter revolver is \odot , and the lense slider is shifted to the right. Press or pull the ocular to focus the reticle. Then adjust the telescope to infinity by autocollimation: Place the mirror in the grating holder and align the telescope approximately perpendicular to the mirror.

Illuminate the reticule with the GAUSS ocular. The light is reflected on the mirror, and you can see the bright circular area of the telescope itself in the telescope. Find the black reflection of the bright illuminated reticule and bring it into focus. If you see it sharply, the telescope is adjusted to infinity.

Collimator: Remove the mirror. Place the telescope opposite to the collimator (they share now the same optical axis) and lock it. You should see the slit through the telescope now. Do not readjust the telescope at this point! If the slit is not sharp, carefully shift the whole slit into or out of the collimator in order to focus it. Adjust the optimum slit width: As small as possible, but slit and reticule good visible.

Lock the telescope.

Grating: Adjust the grating perpendicular to the common axis of the telescope and collimator. For this purpose use the mirror again and illuminate the reticule as above. Bring the black reflex of the reticule in coincidence with the bright reticule. Now the telescope is exactly perpendicular to the mirror.

When all adjustments are done, lock the grid table, turn the light of the GAUSS ocular off, replace the mirror by the grating, and unlock the telescope.

4.2 For measuring the diffraction angle φ_k , bring the reticule in coincidence with the spectral lines and read the corresponding angles φ' . You have to measure 6 spectral

lines in the first, second and third diffraction order, respectively, on the left - as well as on the right side relatively to the diffraction order zero. The diffraction angles then follow from:

$$\varphi_k = \frac{\varphi'_{right} - \varphi'_{left}}{2} \quad (8)$$

5 Evaluation

Calculate the diffraction angles φ_k and then the wavelengths λ by means of eq. (8) and (5), respectively.

Plot the wavelength versus the diffraction angle (the dispersion curves) for each diffraction order in a diagram (all three curves in one diagram).

Compare your results with the values given in tables. If the measurements have been done exactly, the deviations from table values should be not much larger than the estimated uncertainties.

6 Questions

6.1 Which kind of interferences occur on an optical grating?

6.2 For what is a diffraction spectrometer used? How does it work?

6.3 Explain the (wavelength) resolution of a diffraction spectrometer.

Polarimeter and Refractometer

O 10

1 Tasks

1.1 Determine the concentration of a sugar solution by means of a polarimeter.

1.2 Measure the refractive index of glycerol-water mixtures in dependence on the glycerol concentration using a refractometer.

1.3 Determine the concentration of a given glycerol-water mixture.

2 Physical Basis

2.1 Light waves belong to the electromagnetic waves. Each light beam consists of a vast number of separate wave trains. A wave train consists of an electric and a magnetic field which are both perpendicular to the direction of propagation and perpendicular to each other, see fig.1.

If we consider natural, i.e. unpolarized light, the electric and magnetic fields can vibrate in arbitrary directions which, however, are always perpendicular (transversal) to the direction of propagation.

Light is linearly polarized if all electric fields vibrate in only one transversal direction. The direction of the electric field-strength vector

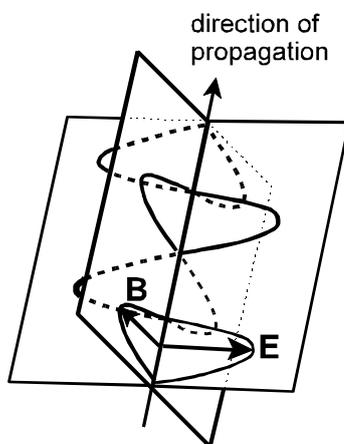


Fig.1: Position of electric and magnetic field-strength vectors for a wave train

is then called the direction of oscillation or the polarizing direction.

2.2 Linearly polarized light may be generated from natural light by (a) reflection at the BREWSTER angle, (b) by birefringence (double refraction in a NICOL prism) or (c) by means of polarizing filters on the basis of diroitic foils.

Optically active materials are substances that rotate the direction of oscillation when linearly polarized light passes the substance. This optical activity may be caused by asymmetric molecule structures or by a screw-like arrangement of the lattice elements. Some substances like sugar have both a dextro-rotatory and a laevorotatory version.

In solutions of optically active substances the angle of rotation depends on the kind of substance, the thickness of the layer penetrated by the light (i.e. the length l of the polarimeter tube) and on the concentration c of the substance. Furthermore, there is a wave-length dependence called rotary dispersion: blue light is stronger rotated than red one. This effect is not considered here.

It applies for the rotation angle φ :

$$\varphi = k \cdot l \cdot c, \quad (1)$$

where the material constant k is called specific rotary power.

2.3 The refractive index n of a substance is defined as the ratio of the vacuum velocity of light to the velocity of light in the substance:

$$n = \frac{c_0}{c}. \quad (2)$$

The refractive index depends on the material and on the wave length of light (this effect is called dispersion). In a solution it also depends on the concentration (mixing ratio). Therefore, a measurement of the refractive index may be suitable for determining con-

centrations.

Applications, for example, are the determination of the protein content in a blood serum or of the sugar degree of grape juice in a winery.

During the transition of light from an optically thinner medium with index n_1 to an optically denser medium with index n_2 ($n_2 > n_1$) a light beam is refracted towards the perpendicular of incidence, see fig.2. With α and β as angle of entry and emergence, the law of refraction reads

$$n_1 \cdot \sin \alpha = n_2 \cdot \sin \beta . \quad (3)$$

For the largest possible angle of entry $\alpha = 90^\circ$ (striping light entry) a maximum refraction angle β_{max} can be obtained.

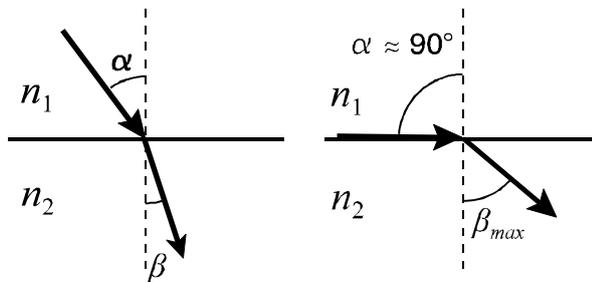


Fig.2: Ray trace of refraction for $n_2 > n_1$.
Left: general case, right: striping light entry

The path of rays in fig.2 can be inverted: From the optically denser medium (n_2) to the optically thinner medium (n_1), angle of entry β , angle of emergence α . For $\beta > \beta_{max}$ no light will be refracted into the optically thinner medium because the law of refraction cannot be fulfilled. Instead, the light is completely reflected at the interface of the two media. Therefore, β_{max} is called critical angle of total reflection. It results from eq. (3):

$$\sin \beta_{max} = \frac{n_1}{n_2} . \quad (4)$$

If the refractive index n_2 (measuring prism of refractometer) is known, the refractive index n_1 of the other medium can be determined by measuring the critical angle of total reflection.

Therefore the interface is lighted via a frosted glass plate with a rough surface, see fig.3. In this way the light beams enter at the interface from all angles between 0° and 90° . So all refraction angles between 0° and β_{max} are possible. When looking at the interface through a telescope at the angle β_{max} , a light-dark boundary can be seen which is used to determine the refractive index of the substance under investigation (as described below).

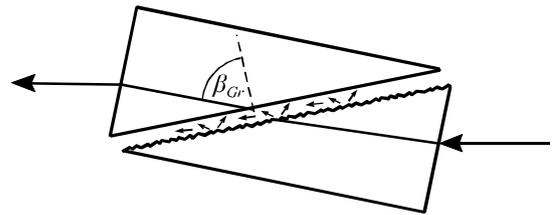


Fig.3: Ray trace at an ABBE refractometer

3 Experimental Setup

3.0 Devices

- polarimeter with sodium-spectral lamp
- polarimeter tube (length 200 mm)
- flask with sugar solution
- ABBE refractometer
- 2 burettes with glycerol 83 vol% and deionized water
- 3 beakers, funnel, pipette
- flask with a glycerol-water mixture of unknown concentration

3.1 The polarimeter consists of a monochromatic light source (Na-D light, $\lambda = 589.3$ nm), the polarimeter tube, polarizer and a rotary analyser with an angular scale.

If the polarizing directions of polarizer and analyser are perpendicular to each other (“crossed position”), no light is transmitted and the visual field of the polarimeter is dark. After placing the polarimeter tube filled with an optically active medium between polarizer and analyser, the visual field is brightened because the direction of oscillation of the linearly polarized light has been rotated by an

angle φ . Resetting the analyser by this angle yields the visual field becoming dark again. In this way the angle φ can be measured. An adjustment of the polarimeter to maximum darkness or brightness without any visual comparison would be imprecise. Therefore, a three-part polarizer is used, resulting in a visual field according fig.4. The inner part of the polariser is tilted against the outer parts by 10° . During the measurement the analyser is adjusted to equal brightness of all three parts in the visual field (half-shade polarimeter). For a precise measurement of angles, the scale is equipped with a vernier that allows a read off with an uncertainty of only 0.05° .

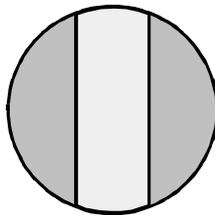


Fig.4: Three-part visual field of the polarimeter

3.2 The ABBE refractometer measures the refractive index n_D (for Na-D light, 589 nm) using natural white light. The dispersion which is quite strong for liquids is compensated by a pair of AMICI prisms with direct vision at 589 nm. Other refractometers are working with monochromatic light. Such instruments would not need compensation of dispersion but they are not able to measure it. The device used consists of the following essential parts:

- the lightning prism with a rough surface
- the measuring prism whose refractive index n_2 must be larger than the refractive index n_1 of the substance under investigation
- a tilting telescope for observing both the measuring prism and an angular scale calibrated according to eq. (4) for reading the refractive index
- an equipment for compensating (or measur-

ing) the dispersion

The grazingly incident part of light ($\alpha \approx 90^\circ$) is refracted at the critical angle β_{max} and can be observed in the telescope as a light/dark boundary.

A simple check of the device can be made by measuring the refractive index of pure water, which is $n_D = 1,3330$ at 20°C and $1,3325$ at 25°C .

4 Experimental Procedure

4.1 Switch the sodium spectral lamp on at first; it needs about 5 minutes to reach its maximum brightness.

Determine the zero position φ_0 of the polarimeter by adjusting the visual field as described in 3.1, but without polarimeter tube. Take the reading of φ_0 5 times and readjust the polarimeter for every reading.

If necessary clean the glass windows of the polarimeter tube. The windows can be easily removed from the screw caps. When screwing the cap onto the tube, assure the rubber O-ring is between glass window and metal cap (not between window and glass tube). Do not tighten it too much!

Fill the polarimeter tube completely with sugar solution. You may fill it on the washstand and dry it with paper towels. There must be no bubble in the beam path. A remaining small bubble may be set into the bulge of the tube. Finally, put the tube into the polarimeter.

Now adjust the analyser again to equal brightness of the visual field, and read the corresponding angle φ_1 . This measurement is to be carried out 5 times, too.

Then the rotation angle results as the difference of the mean values of φ_1 and φ_0 :

$$\varphi = \overline{\varphi_1} - \overline{\varphi_0} . \quad (5)$$

When finished, fill the sugar solution back to the flask. Clean the polarimeter tube with water and leave it open.

4.2 The two prisms of the refractometer must be on the right side, and the small

mirror for scale illumination (left hand side) must be open.

Open the two prisms (measuring prism above and lightning prism below). If necessary clean the prisms carefully with wet paper towel and dry it. Hold the lightning prism with the rough surface about horizontally and put 1 or 2 drops of the sample liquid on the surface. Ensure that there are no air bubbles in the liquid. Then close the prisms and lock them gently.

Look through the measuring eyepiece (the right one) and adjust it until the reticle is sharply seen. Adjust the lightning mirror for maximum lightness. By turning the scale adjustment knob (on the left) move along the measuring range until the light/dark boundary appears. Eliminate colour fringes by turning the compensation knob (on the right) until the boundary appears black-and-white. Adjust the centre of the reticle exactly to the light/dark boundary and read the corresponding refractive index on the scale (left eyepiece).

Before taking measurements, check the adjustment of the device with de-ionized water. If the reading differs by more than one scale division from the reference value, ask the tutor to adjust the refractometer.

The refractive index is to be determined for the following liquids:

- de-ionized water
- glycerol 83 vol%
- 5 glycerol-water mixtures:
4:1, 4:2, 4:4, 4:8, 4:16 and
- a glycerol-water mixture of unknown concentration

For the mixture 4:1 take 4 ml glycerol 83% and 1 ml deion. water, and make the other mixtures by further dilution of this mixture with water.

Measure each refractive index 5 times (re-adjust the scale for each measurement).

Clean the prisms when changing the concentration and at the end of the measurements.

4.3 The refractive index of the glycerol-water mixture of unknown concentration is to be measured 5 times as well.

5 Evaluation

5.1 Calculate the concentration c (in g/l) of the sugar solution according to the equations (1) and (5).

The specific rotary power of saccharose ($C_{12}H_{22}O_{11}$) at $\lambda = 589.3$ nm amounts to $k = 0.66456$ deg l m⁻¹ g⁻¹. The length of the polarimeter tube is $l = (200 \pm 0.2)$ mm.

Calculate the uncertainty of the concentration using Gauss's error propagation law. $u(\varphi)$ has to be calculated from the standard deviations of the means $\bar{\varphi}_0$ and $\bar{\varphi}_1$.

5.2 Calculate the volume concentrations (pure glycerol in water) of all mixtures. Plot the refractive index via the volume concentration of glycerol in water.

5.3 Determine the concentration of the unknown glycerol-water mixture by means of the diagram from 5.2.; give the concentrations in terms of vol.% glycerol.

6 Questions

6.1 What is light?

6.2 How can linearly polarized light be generated?

6.3 What is refraction? When does total reflection occur?

6.4 Which influence has the dispersion on measurements with a refractometer?

Radioactivity

O 16

1 Tasks

1.1 Measure the dependence of nuclear radiation on the distance to the radiation source and verify the inverse-square law.

1.2 Determine the attenuation coefficient and the half-value thickness (HVT) of lead (Pb) for the gamma radiation of Co-60.

1.3 Investigate the frequency distribution of the counts (counting statistics).

2 Physical Basis

Radioactivity is a property of atomic nuclei having unfavourable proton-neutron ratios. Such nuclei transform spontaneously by emission of characteristic radiation into other atomic nuclei or into nuclei of another energy level (they are said to decay). Depending on the kind of transformation, the radiation consists of particles and high energetic electromagnetic waves:

α particles = He nuclei (2 protons, 2 neutrons),

β^- particles = electrons,

β^+ particles = positrons,

γ quanta (electromagnetic radiation with a quantum energy >100 keV),

neutrons and (rarely) protons.

γ radiation arises when after a nuclear transformation the excited nucleus returns into its basic energy level.

2.1 The number of nuclei transforming in a time interval is proportional to the total number of nuclei being present. The number of decays per time within a sample of radioactive material is called the activity A :

$$A = -\frac{dN}{dt}. \quad (1)$$

After the time interval dt the number of

radioactive nuclei is lowered by

$$dN = -\lambda \cdot N \cdot dt. \quad (2)$$

λ is called the radioactive decay constant. From eq. (2) follows the law of radioactive decay:

$$N(t) = N_0 \cdot e^{-\lambda \cdot t} \quad (3)$$

with N_0 being the number of radioactive nuclei at the time $t = 0$.

If a γ quant (or α or β particle) is detected by a Geiger-Müller tube (GM tube), it triggers a current pulse. The pulses are counted, and the pulse rate I (the number of pulses per second) is proportional to the radiation intensity. Additionally, it depends on the characteristics of the detector and possibly on the energy of the radiation.

The pulse rate I caused by a radioactive preparation is the difference of the pulse rates measured with preparation I_p and without preparation I_z (zero rate):

$$I = I_p - I_z. \quad (4)$$

The zero rate is caused by environmental radiation (cosmic radiation and natural radioactivity) and by interfering pulses of the detector.

2.2 If gamma radiation penetrates matter, its intensity (measured as pulse rate I) reduces depending on the penetrated thickness x according to the attenuation law

$$I = I_0 \cdot e^{-\mu \cdot x}. \quad (5)$$

Here, I_0 is the intensity of the incident radiation and I the intensity of the escaping radiation; μ is called the attenuation coefficient, it depends on the material penetrated and on the energy of the gamma quanta.

Besides elastic scattering (μ_s), three different absorption effects are responsible for the attenuation: the photo effect (μ_{ph}), inelastic

scattering (Compton effect, μ_C) and the pair creation effect (μ_p):

$$\mu = \mu_S + \mu_{Ph} + \mu_C + \mu_p . \quad (6)$$

The portion of these effects on the total attenuation depends on the energy. At low energy elastic scattering predominates, and at very high energy the pair creation is dominant.

The half-value thickness (HVT) $x_{1/2}$ of a material is the thickness required for the intensity to be attenuated to its half value. From eq. (5), it follows for $I = \frac{1}{2} I_0$:

$$x_{1/2} = \frac{\ln 2}{\mu} . \quad (7)$$

2.3 The radioactive decay of a nucleus is a quantum process. The prediction of the exact time of a decay is in principle impossible. Only the probability of the nucleus to decay in a certain time interval is known. Therefore the number of counts measured is for fundamental reasons (and not only because of the measurement errors of the devices used) a random number. This is particularly noticed when the counts measured are low.

With N being the number of radioactive atoms and p the probability of one atom to decay, the probability of n decays is

$$P_N(n) = \binom{N}{n} p^n (1-p)^{N-n} \quad (8)$$

with the mean value (the expectation) $\mu = n \cdot p$. In our experiment N is a huge number and p is very small. Passing to the limits $N \rightarrow \infty$ and $p \rightarrow 0$, the binomial distribution (8) transforms into a POISSON distribution

$$P(n) = \frac{\mu^n e^{-\mu}}{n!} \quad (9)$$

with the mean value μ . (Pay attention that the mean value μ is not identical with the attenuation coefficient μ in section 2.2!)

An important mathematical property of the POISSON distribution is the equality of mean value μ and variance σ^2 (square of the stan-

dard deviation σ). From that follows:

If a number n of random events is measured in a time period, the uncertainty of the measurement result is

$$\sigma_n = u(n) \approx \sqrt{n} . \quad (10)$$

The statistical uncertainty is approximately equal to the root of the measurement result.

Additionally, for large n the POISSON distribution can be approximated by a GAUSS distribution

$$P(n) = \frac{1}{\sqrt{2\pi\mu}} e^{-\frac{(n-\mu)^2}{2\mu}} . \quad (11)$$

3 Experimental setup

3.0 Devices:

- radioactive preparation Co-60 (γ radiator 1.17 MeV and 1.33 MeV, $A = 74$ kBq 2010, $t_{1/2} = 5.27$ a)
- Geiger-Müller tube
- digital counter
- computer with program "Digitalzähler"
- 50 cm rail with measure
- lead slabs of different thickness

3.1 The GM tube is an end-window counter. It is equipped with a thin mica window that allows also for measuring low energetic γ and X radiation as well as β particles.

The digital counter is both the rate meter and the power supply for the GM tube. The counter automatically stores up to 2000 measured values. For the statistical analysis it sends the counts to the computer.

The radioactive preparation and the GM tube each reside in a plexiglass block mounted on a sledge that can be shifted on the rail with a cm scale. Radioactive preparation and GM tube are facing to each other. A third sledge for carrying the absorbing slabs can be mounted in between them. The distance between the preparation and the centre of the counter tube is 10 mm larger than the distance between their sledges. For adjusting e.g. a

distance of $r = 40$ mm, the distance between the sledges must be set to 3.0 cm.

Radiation Protection:

According to the German Radiation Protection Ordinance, every radioactive exposure, also below the allowed limits, is to be minimised. Therefore: Do not carry the preparation in your hand if not necessary! Keep a distance of 0.5 m to the preparation during the experiment! It is not allowed to remove the Co-60 preparation from its plexiglass block.

4 Experimental procedure

The Co-60 radiator is an enclosed preparation with an activity below the permitted limit according to the Radiation Protection Ordinance. Your radiation exposure in a 3 h lab course is about 0.1% of the exposure caused by a medical radiogram.

4.1 At the digital counter, adjust an operating voltage of 480 V for the GM tube. Choose rate measure, measuring interval 60 s. Display the number of counts N . All measurements shall be carried out five times (5 minutes). The counts are stored every 60 seconds in memory, after stopping the counting you can read the values.

At first, measure the zero rate (five times). Put the preparation at least 1 m away from the counter tube for this measurement. Then place the Co-60 radiator in a distance of 40, 50, 70, 100, 140, 190 and 250 mm from the GM tube and measure the pulse rate for each distance.

4.2 For determining the attenuation coefficient of Pb, put the third sledge between the radiator and the counter tube and place the preparation in a distance of 70 mm from the counter tube. This distance has to be kept constant during the remaining experiment.

Measure the pulse rates for the thicknesses $x = 1, 2, 5, 10, 20$ and 30 mm five times each. The measurement result for $x = 0$ mm can be taken from task 4.1.

4.3 The measurements for the frequency distribution may run unattendedly in the background while you are evaluating other parts of the experiment at the same computer or during a discussion with the tutor.

Delete all previously measured data at the counter and start the program "Digitalzähler". Press [F5] for the options dialogue, select the "Allgemein"-tab and change the language from "Deutsch" to "English". Select "Poisson" from the predefined graph tabs.

Place the radioactive preparation in a distance of 10 cm from the counter tube. Switch the counter to rate measurement with a gate time of 1 s. Start the measurement either at the counter device or at the program and record at least 600 measurements (10 minutes).

Evaluate or save this series of measurements and record a second series with the distance between preparation and counter tube being 5 cm.

5 Evaluation

Calculate the average of the five single measurements in every part of the experiment. Correct the average pulse rates by subtracting the zero rate according to eq. (4).

5.1 The inverse-square distance law is to be verified. (Answer: What's this law?) Plot the pulse rate I versus distance r on double logarithmic scales. Use either double-logarithmic graph paper or a computer in the lab to do this. Fit a straight line to the measuring points and determine the slope. The slope s is the exponent in a distance law of the kind $I(r) = C \cdot r^s$. Compare your result with the theoretical distance law.

5.2 By taking the logarithm of eq. (5) we get

$$\text{either } \ln I = \ln I_0 - \mu \cdot x$$

$$\text{or } \lg I = \lg I_0 - \mu \cdot x \cdot \lg e$$

with $\lg e = 1/\ln 10 = 0.434$. For determining the attenuation coefficient μ of lead, plot the pulse rates I versus the total thickness x of the absorbing Pb slabs on single logarithmic coordinates (rate logarithmic, thickness linear). Alternatively, you can calculate the logarithm of the rate $\ln(I)$ or $\lg(I)$ and plot it versus thickness on linear (“normal”) scales. (Although taking the natural logarithm is easier here, in common scientific praxis the decimal logarithm is preferred because the graph is better readable.)

In both cases the measuring points should follow a straight line. Calculate the attenuation coefficient μ from the slope of this line. With the knowledge of μ , calculate the HVT.

5.3 For the two series of measurements calculate the mean value \bar{n} and the standard deviation σ . Check whether the prediction $\sigma \approx \sqrt{\bar{n}}$ is valid.

Plot the frequency distributions as bar graph and (in the same graph) the Poisson distribution and the normal distribution fitting the data as curves.

These tasks are easily done with the program “Digitalzähler”. Look for the menu item “Fit function” in the context menu of the graph.

6 Questions

6.1 What is the difference between X-rays and γ -rays?

6.2 What is the half value thickness and the half life period?

6.3 How does the intensity of radiation depend on the distance from the radiator?

6.4 A counter tube measures 10 000 pulses. How large is the uncertainty of this measurement?

X-ray methods

O 22

1 Tasks

1.1 Measure the X-ray emission spectra of a molybdenum anode using a LiF crystal and determine the maximum quantum energy of the X-radiation in dependence on the anode voltage.

1.2 Determine the ion dose rate of the X-ray tube within the apparatus.

1.3 X-ray examination and interpretation on several objects (bones, computer mouse, ...).

2 Physical Basis

2.1 X-ray radiation

X-rays are electromagnetic waves (photons) with wavelengths between 0,01nm and 10nm. They are produced by bombarding an anode with electrons the energy of which exceeds 10 keV. At the impact two types of X-ray radiation are produced besides approx. 98% of heat:

(i) Bremsstrahlung is produced by the sudden slowing down of incident electrons in the vicinity of the strong electric field of the atomic nuclei of the anode material. After this interaction the electrons still have a part of their kinetic energy. The difference between the kinetic energy before and after the interaction is transformed into X-rays with the frequency f . (see equation (2))

With E being the kinetic energy of the electrons after acceleration through the voltage U , the following energy balance results:

$$E = e \cdot U = \frac{m_e}{2} v_1^2 = E_{Ph} + \frac{m_e}{2} v_2^2 \quad (1)$$

with

e : elementary charge of the electron ($e=1,602 \cdot 10^{-19}$ C)

U : anode voltage

m_e : electron mass

v_1 : velocity of the electron before the impact

v_2 : velocity of the electron after the impact

E_{Ph} : photon energy (energy of an X-ray quantum)

The energy of a radiation quantum is

$$E_{Ph} = h \cdot f = h \cdot \frac{c}{\lambda} \quad (2)$$

h : PLANCKS constant ($h = 6,625 \cdot 10^{-34}$ Ws²)

c : velocity of light in vacuum

($c = 2,998 \cdot 10^8$ m s⁻¹)

f : frequency

λ : wavelength

The bremsstrahlung has a continuous spectrum with an edge at short wavelengths (see fig.1). This corresponds to those electrons which transpose their whole kinetic energy into an X-ray photon (total slowdown, $v_2=0$). The photon has then a maximal energy, hence its wavelength is minimal in this case:

$$E_{Ph} = e \cdot U = h \cdot f_{max} = h \cdot \frac{c}{\lambda_{min}} \quad (3)$$

The energy in that context is usually counted in eV (electron volts). 1 eV is the energy that a particle with one elementary charge e gets when accelerated through a voltage of 1 V. The energy in Joule is hence calculated by multiplying the eV with $e = 1.602 \cdot 10^{-19}$ As.

(ii) Characteristic radiation: During the impact of electrons, atoms of the anode material are ionised. If due to this a vacancy in the innermost shell - the K-shell - arises, it will be immediately occupied by L- and M-electrons, respectively, and the energy differences will be released in form of X-rays. The photons (energy quanta) which are emitted during these electron jumps are called K_α and K_β photons, respectively. The corresponding wavelengths can be calculated from

$$\lambda_{K_\alpha} = \frac{h \cdot c}{E_L - E_K} \quad \lambda_{K_\beta} = \frac{h \cdot c}{E_M - E_K} \quad (4)$$

$E_L - E_K$: the difference in electron energy between the L- and K-shell

$E_M - E_K$: the difference in electron energy between the M- and K-shell

Because this energy difference is characteristic of the material, the radiation is called „characteristic radiation”. This radiation exhibits a line spectrum.

Fig. 1 shows a typical X-ray spectrum consisting of Bremsstrahlung and characteristic radiation. The spectrum of the Molybdenum anode used in this experiment has a similar shape.

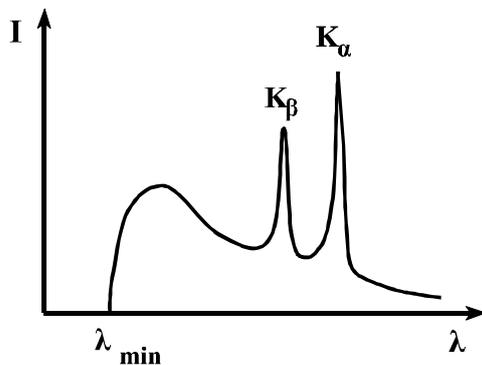


Fig.1: typical X-ray spectrum revealing Bremsstrahlung and characteristic radiation

X-ray diffraction:

The wave length of X-rays may be determined by means of diffraction on a crystal lattice when the lattice distances are known (X-ray spectral analysis). Inversely, with X-rays of known wavelength the lattice distances of crystals may be determined (X-ray diffraction analysis, BRAGG's method).

According to the HUYGENS principle, each atom of the crystal hit by X-rays can be considered as a source of an elementary wave. The atoms in the crystal can be summarized in multiple consecutive layers situated parallel to the crystal surface. This planes are called "lattice planes". In the simplest case the diffraction of X-rays can be described as reflection at the lattice planes of

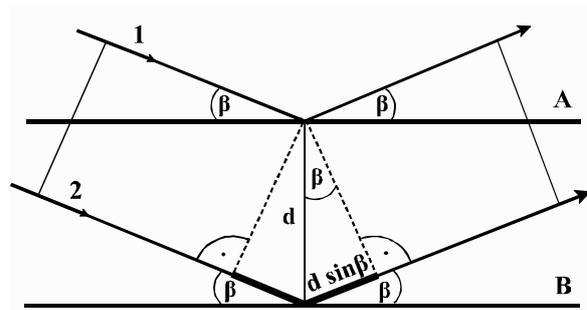


Fig. 2 BRAGG reflexion

a crystal. Each lattice plane acts on the incident X-ray like a partial mirror, that reflects a (very small) part of the incident X-ray.

Fig. 2 shows the fundamental processes of this so-called "BRAGG reflection": The rays 1 and 2 reflected on the planes A and B interfere with each other. Constructive interference (a so-called "reflex") appears only when the path difference $2 d \cdot \sin \beta$ between the two waves equals a multiple of wavelengths:

$$2 \cdot d \cdot \sin \beta = k \cdot \lambda, \quad k = 1, 2, 3, \dots \quad (5)$$

k is the order of diffraction and d is the lattice constant ($d = 0,201$ nm for the LiF crystal used in that experiment). For the first order of diffraction ($k=1$), from equation (2) follows:

$$E_{ph} = \frac{h \cdot c}{2 d \sin \beta} \quad (6)$$

By rotating the crystal the incidence angle of the X-rays β and thus the path difference of the interfering rays can be varied so that the condition for constructive interference (5) can be fulfilled for different wavelengths of the primary rays, respectively. While rotating the crystal, also the radiation detector has to be maintained at the Bragg angle, so that the reflection condition *detector angle* = $2 \times$ *crystal angle* is always fulfilled. In this way the spectrum of the X-ray source can be determined.

2.2 Dosimetry is the measurement of the impact that ionising rays (X-rays and radioactive rays) do have on matter. This impact can be measured in two ways: by measuring the number of ions created within the matter or by measuring the amount of energy absorbed

by the matter.

The ion dose J is defined as the total charge of ions ΔQ produced in a volume element divided by the mass Δm of that volume element:

$$J = \frac{\Delta Q}{\Delta m}. \quad (7)$$

The unit of measure of the ion dose is As/kg or C/kg.

The absorbed dose D is defined as the energy ΔE absorbed by a volume element divided by the mass of the radiated volume element Δm :

$$D = \frac{\Delta E}{\Delta m}. \quad (8)$$

Its unit of measure is the Gray (Gy), 1 Gy = 1 J/kg.

The equivalent dose H characterises the biological impact of ionising radiation and is defined as

$$H = w \cdot D \quad (9)$$

with the unit Sievert (Sv), 1 Sv = 1 J/kg. w is the radiation weighting factor, it is $w = 1$ for X-ray, gamma and beta rays and $w = 20$ for alpha rays.

The effective intensity of ionising rays is the dose per time or dose rate. It may be given as ion dose rate j (in A/kg), absorbed dose rate d (Gy/s) or equivalent dose rate h (Sv/s). 1 Sv/s is a very large unit (6 Sv are lethal to humans), therefore mSv/h or μ Sv/h are more common units.

The ion dose rate is usually measured with an ionisation chamber, that is in principle a large capacitor filled with air of the mass m as shown in fig.3. A voltage is applied to the capacitor that is large enough for all ions to get to the plates. The radiation causes an ion current I_C that can be measured in the outer circuit. The ion dose rate is than

$$j = \frac{\Delta Q}{\Delta m \Delta t} = \frac{I_C}{m}. \quad (10)$$

With the known mean ionisation energy of air

molecules the equivalent dose is calculated from the ion dose according to

$$H = J \cdot 32,5 \frac{\text{Sv}}{\text{As kg}^{-1}} \quad \text{or} \quad (11)$$

$$h = j \cdot 32,5 \frac{\text{Sv}}{\text{As kg}^{-1}}.$$

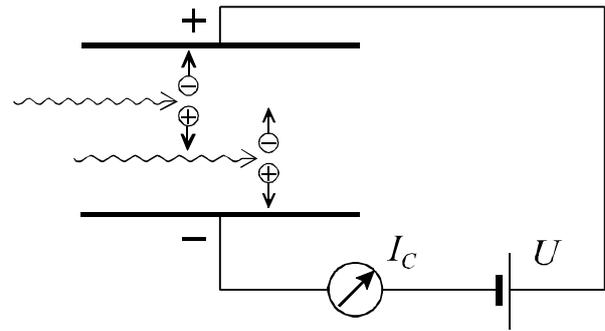


Fig.3: Measurement of the ion dose rate in an ionisation chamber

3 Experimental setup

3.0 Devices

- X-ray device with goniometer including LiF crystal ($d = 0,201 \text{ nm}$) and G.M.-counter.
- PC with program “Röntgengerät”
- capacitor with X-ray aperture for ion dose measurements (build into X-ray device)
- power supply 0...450 V, $R_i = 5 \text{ M}\Omega$
- measurement amplifier
- electrical multimeter
- cables
- several objects for X-raying.

3.1 The X-ray device (see fig.4) consists of a radiation shielding case that is separated into three chambers. The largest (right-hand side) chamber is the experimental chamber. It contains either the goniometer (for diffraction measurements) or the capacitor (for dose measurements) or the objects for X-raying. The X-ray tube is placed in the middle chamber. The left chamber contains the microprocessor controlled electronics, the controls and displays.

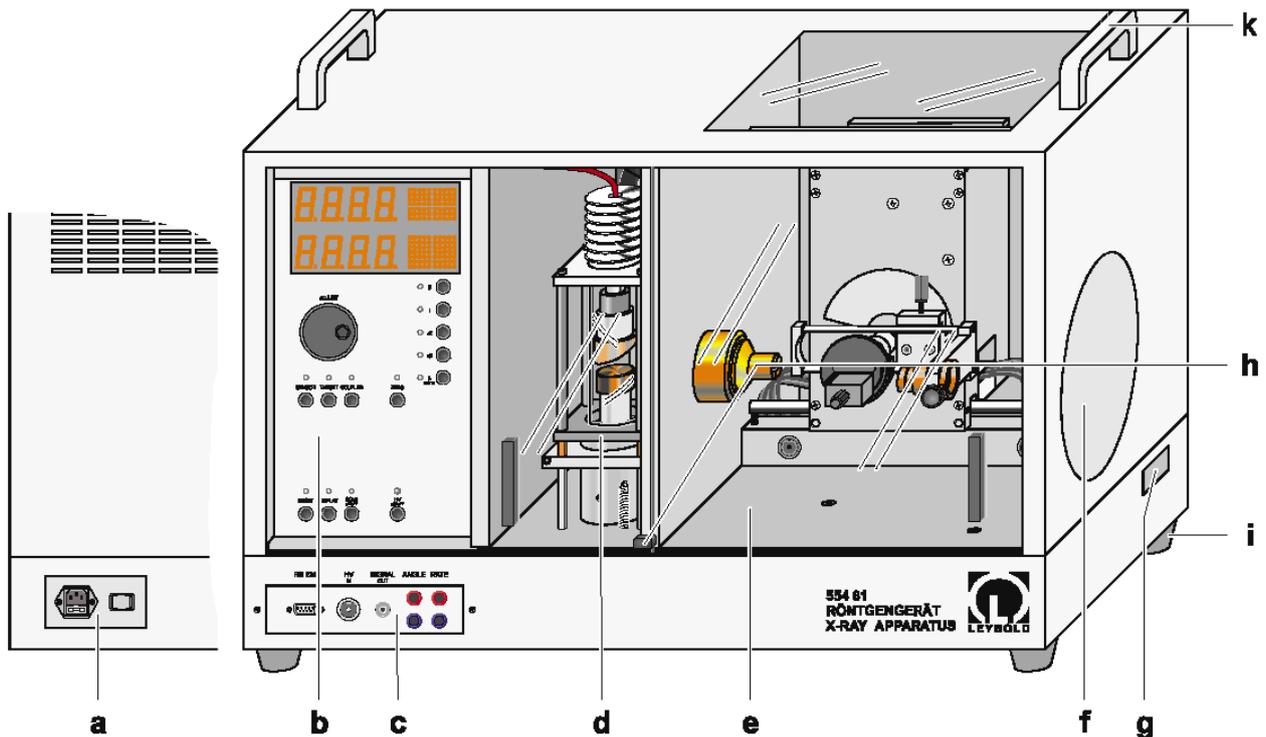


Fig.4: X-ray device with goniometer.

a Mains power panel, **b** Control panel, **c** Connection panel, **d** Tube chamber (with Mo tube), **e** Experiment chamber with goniometer, **f** Fluorescent scree, **g** Free channel, **h** Lock lever

The doors and windows consist of lead glass. This is a very soft material! Handle with care, do not scratch it!

3.2 The high voltage power supply exhibits a very large output resistance. The contacts may be touched without harm. For measuring the very small current an amplifier and the multimeter are used.

Security declaration:

The device is constructed in a manner that X-ray is only created when the doors of the chambers are closed. The radiation outside of the case falls several times off the admissible limit according to the German Radiation Protection Ordinance. According to the "Verordnung über den Schutz vor Schäden durch Röntgenstrahlen" the X-ray device is an admitted model. (admission symbol NW807/97Rö)

4 Experimental procedure

Please do not touch the LiF crystal fixed on the goniometer.

4.1 Use the X-ray device with the build-in diffractometer. Set up the following parameters for recording the X-ray spectra in the BRAGG arrangement:

Tube current:	$I = 1,0 \text{ mA}$
High voltage:	$U = 20 \dots 35 \text{ kV}$
Measuring time:	$\Delta t = 5 \text{ s}$
Step width:	$\Delta\beta = 0,1^\circ$
goniometer mode:	coupled
Initial angle:	$\beta_{\min} = 4,0^\circ$
Final angle:	$\beta_{\max} = 12,0^\circ$

Start the computer program "Röntgenerät". You may change the program language from German to English (press F5, choose Allgemein and change Sprache).

The best way is to start with the maximum acceleration voltage (35 kV). The recording

is started by pressing the SCAN button at the X-ray device. Record additional spectra at 30 kV, 25 kV and 20 kV into the same graph. To increase the accuracy of the measured values at low acceleration voltages, you can increase the measuring time Δt . To save time you can reduce the measuring range (increase β_{\min}) as long as the edge of the spectrum is just in the measuring range.

4.2 Use the X-ray device with the build-in capacitor for the ion dose measurement. Complete the wiring according to figure 3: Connect the coaxial cable from the lower capacitor plate to the current input I of the amplifier. Interconnect the ground socket (\perp) of the amplifier with the negative terminal and the upper capacitor plate with the positive terminal of the power supply. Connect the multimeter to the output of the amplifier and select the range 10^{-9} A (1 V output is equivalent to $I_C = 1$ nA).

Measure the ion current I_C at the maximum acceleration voltage of 35 kV and with tube currents of 1 mA, 0.8 mA, 0.6 mA, 0.4 mA and 0.2 mA. Record the air pressure p and the temperature T in the X-ray device.

4.3 For X-raying of objects use the particular X-ray device prepared for this task. Adjust the maximum possible energy ($U = 35$ kV, $I = 1$ mA). The room has to be darkened.

Observe the shade of the object under investigation on the screen. Investigate how the image depends on the position of the object in the chamber.

X-ray the objects given and objects you own (pocket calculator, ball pen, ...) and record the observations to your protocol.

After finishing this part of the experiment, the fluorescent screen has to be covered again.

5 Evaluation

5.1 Determine the wavelength and quantum energies for the characteristic lines K_α and K_β of the Mo anode, using equation (5) and (6), respectively. The quantum energies are usually given in keV.

Calculate the maximal quantum energy for each value of the anode voltage U from the angles β of the corresponding short-wave edge, using equation (6). List the energies in a table and compare them with the kinetic energy $E = e \cdot U$ of the electrons accelerated by the voltage U .

As part of your consideration of errors, estimate the wave length resolution of the X-ray device.

5.2 Calculate the ion dose rate according to (10) from the ion current I_C and the mass m of the radiated air volume. This mass is given by

$$m = \rho \cdot V, \quad \rho = \rho_0 \cdot \frac{T_0 p}{T p_0} \quad (12)$$

with $V = 125 \text{ cm}^3$, $\rho_0 = 1,293 \text{ kg/m}^3$,
 $T_0 = 273 \text{ K}$ and $p_0 = 1013 \text{ hPa}$.

Additionally, calculate the maximum equivalent dose within the X-ray device at $I=1\text{mA}$, in units of Sv/h by means of eq. (11) and compare it with other values like the annual natural dose and the lethal dose.

5.3 Record the observations made during the X-ray screening of the objects in your protocol.

6 Questions

6.1 Explain the spectrum of an X-ray tube. How is it influenced by tube current and acceleration voltage?

6.2 How is the biological effect of ionising radiation measured?

6.3 Which X-ray methods for material investigation do you know?

Physical constants

velocity of light in vacuum	c	=	$2,997\,924\,58 \cdot 10^8$ m/s
		≈	300 000 km/s
gravitational constant	γ	=	$6,674\,1(3) \cdot 10^{-11}$ N m ² kg ⁻²
gravity acceleration (local value)	g	=	9,812 03(2) m/s ²
elementary charge	e_0	=	$1,602\,176\,62(1) \cdot 10^{-19}$ C
electron mass	m_e	=	$9,109\,383\,6(1) \cdot 10^{-31}$ kg
atomic mass unit	u	=	$1,660\,539\,04(2) \cdot 10^{-27}$ kg
electric field constant (dielectric constant of free space)	ϵ_0	=	$8,854\,187\,817 \cdot 10^{-12}$ A s V ⁻¹ m ⁻¹
magnetic field constant (permeability of free space)	μ_0	=	$4 \pi \cdot 10^{-7}$ V s A ⁻¹ m ⁻¹
		≈	$1,256\,637 \dots \cdot 10^{-6}$ V s A ⁻¹ m ⁻¹
Planck constant (quantum of action)	h	=	$6,626\,070\,04(8) \cdot 10^{-34}$ J s
		=	$4,135\,667\,66(3) \cdot 10^{-15}$ eV s
Avogadro constant	N_A	=	$6,022\,140\,86(8) \cdot 10^{23}$ mol ⁻¹
Boltzmann constant	k	=	$1,380\,648\,5(8) \cdot 10^{-23}$ J/K
molar gas constant	R	=	8,314 460(5) J mol ⁻¹ K ⁻¹
Faraday constant	F	=	$9,648\,533\,29(6) \cdot 10^4$ C/mol

Source and more data: <http://physics.nist.gov/cuu/>